

話題・解説 (III)

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- Prediction of fission observables with Langevin equation. -

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1. Introduction

In recent years, considerable improvements to the predictions of fission observables from the calculation of Langevin equation have been observed. For many years, this calculation are often done with respect to three collective variables and such calculations are often termed 3 dimensional (3D) Langevin calculation. Additionally, most Langevin calculations are done with respect to macroscopic transport coefficients [1–5]. In recent years, we at Chiba Laboratory have developed microscopic transport coefficients for use with 3D-Langevin calculation [6, 7] and we have also developed 4D-Langevin calculation [8].

Let us begin by examining how one of the fission observables, the primary fission fragment yields are often predicted by Langevin equation. Primary fission fragment yields are calculated by taking account of the statistics of the fission fragment mass. Assuming the case of binary fission in Langevin equation, the masses of the fission fragment m_i for each fragment $i = \{1, 2\}$ are taken into account through the collective variable $\alpha = (m_1 - m_2)/(m_1 + m_2)$. In practice, for a given nucleus shape we only need to calculate the volume, V of each fragment and calculate $\alpha = (V_1 - V_2)/(V_1/V_2)$ by assuming that the volume is directly proportional to the mass.

In our calculation of the Langevin equation, this collective variable evolves in time from compound nucleus to scission. Scission in our calculation is defined to be the moment when the radius of the neck connecting the two fragments becomes zero. Hence, it should be safe to assume that the fission fragment mass at scission will stay the same until the point of detection. As we accumulate more fission events, we will be able to obtain the statistics to compare the yield from our calculation with similar data from either experimental or evaluated efforts.



Fig. 1: Two centre shell model profile and nuclear potential.

Apart from α , other collective variables are based on the two centre shell model [9] shape parameterizations and depicted in Fig. 1. The two centre shell model attempts to describe each fragment with a harmonic potential and then fit the neck with a quadratic function and will be described further later on. Based on this model, we are able to define all the collective variables used in our current 4D and 3D-Langevin calculation,

$$q_{\mu}^{4\mathrm{D}} = \left\{ \frac{z_0}{R_0}, \delta_1, \delta_2, \alpha \right\}, \qquad q_{\mu}^{3\mathrm{D}} = \left\{ \frac{z_0}{R_0}, \delta, \alpha \right\}.$$
(1)

The difference of 4D-Langevin with 3D-Langevin calculation, lays only the requirement that δ_1 must always be equal to δ_2 in 3D-Langevin, hence we write $\delta_2 = \delta_1 = \delta = 2$. In 4D-Langevin, δ_1 and δ_2 are allowed to evolve completely independent of each other. Here $\delta_i = \frac{3(a_i - b_i)}{(2a_i + b_i)}$ measures the deformation of the fragment tips. Negative δ_i indicates oblate fragment shape, $\delta_i = 0$ means the fragment shape is perfectly spherical and positive δ_i implies that the fragment shape is prolate.

At each set of collective variables, the nucleus will possess a shape that will allow us to obtain other pertinent information such as the distance between the centres of masses for the two fragments R_{12} . In our preceding discussion, our focus is with regards to 3D-Langevin calculation.

At scission, the value of R_{12} at that moment allows us to easily calculate by point approximation the amount of kinetic energy possessed by the fission fragments that are speeding away from each other due to Coulomb force. If we also add the speed at which the fragment separates just prior to scission, the total kinetic energy (TKE) of the fission event is obtained.

Hence, it is important to realize that several other models are able to study the same fission observables if the same collective variables are being used. For example, in the scission point model [10], a chosen set of scission points or selected by optimal shapes [11,12] and then corrected for excitation energy [13] may also give similar fission observables.

The advantages then of handling these collective variables through the Langevin equation could be maximized by taking full advantage of the dynamical nature of the equation. At excitation energy just above neutron separation, perhaps only a single saddle point will lead to a path towards fission if we begin our calculation at the minimum of the potential energy surface. With higher excitation energy, the path through the saddle point widens and possibly even more saddle points opens up. The kinetic energy accumulated and temperature as it passes through the saddle may influence the trajectories towards several different scission configuration. As such, we can predict not just fission observables but also the interplay of the fission observables with as observed in many experiments as shown in some of the results.

2. Langevin Equation

We can write the Langevin equation as a system of first order differential equation governing the evolution of the collective variables q_{μ} and its conjugate momentum p_{μ} ,

$$\frac{dq_{\mu}}{dt} = m_{\mu\nu}^{-1} p_{\nu},$$

$$\frac{dp_{\mu}}{dt} = -\frac{\partial U(q)}{\partial q_{\mu}} - \frac{1}{2} \frac{\partial m_{\nu\sigma}^{-1}}{\partial q_{\mu}} p_{\nu} p_{\sigma} - \gamma_{\mu\nu} m_{\nu\sigma}^{-1} p_{\sigma} + g_{\mu\nu} R_{\nu}(t),$$
(2)

where the summation is performed over repeated indices. The potential energy surface U(q) is taken as the sum of the liquid drop deformation energy E_{LD}^{def} and the shell corrections $\delta E(q)$, namely,

$$U(q, T = 0) = E_{LD}^{def}(q) + \Phi(E_{int})\delta E(q, T = 0).$$
(3)

 $E_{LD}^{def}(q_{\mu})$ is the amount of energy required to deform the spherical liquid drop into liquid drop with shape q_{μ} . The liquid drop energy calculated here is from finite-range liquid drop model [14] with nuclear radius constant $r_0 = 1.20$ fm, Yukawa folding function range a = 0.65 fm, surface energy constant $a_s = 21.836$ MeV and surface-asymmetry constant $\kappa_s = 3.48$. Quantum effects are included into the potential energy surface through shell corrections

$$\delta E(q) = \sum_{n,p} \left(\delta E_{shell}^{(n,p)}(q) + \delta E_{pair}^{(n,p)}(q) \right). \tag{4}$$

 $\delta_{shell}^{(n,p)}(q)$ is calculated as the difference between the sum of single particle energies of the occupied states and its averaged value defined by Strutinsky smoothing [15] and the contributions are summed over neutrons (n) and protons (p). This however doesn't include the effects of pairing on the shell effects. This is mitigated by including the shell correction to the pairing correlation energy $\delta_{pair}^{(n,p)}(q)$ evaluated using BCS theory [16].



(a) Potential energy surface (b) Potential energy surface with liquid drop energy of with shell corrections on the 236 U. liquid drop energy of 236 U.

(c) Shell effects on fission fragments of mass yield 236 U.

Fig. 2: Shell effects on Langevin calculation for fission.

The stark difference is seen between the potential energy surface made out of the liquid drop deformation energy only in Fig. 2a and the potential energy surface which includes shell effects in Fig. 2b. This is reflected in the fission observables as we can see in Fig. 2c for the fission fragments mass yield demonstrating the consequence of shell corrections on the liquid drop energy. In all of our calculations, the shell corrections are calculated by considering temperature, T = 0 MeV.

We could also see in Eq. (3) that the shell correction is controlled by the factor $\Phi(E_{int})$. Two of the more common prescriptions for the shell correction factor are the ones by Ignatyuk [17] written as $\Phi(E_{int}) = e^{-E_{int}/E_d}$ and its extension formulated by Randrup and Moller [18] where $\Phi(E_{int}) = (1 + e^{-E_d/E_0}) / (1 + e^{(E_{int}-E_d)/E_0})$. Here, we often use $E_0 = 15$ MeV and $E_d =$ 20 MeV in our 4D-Langevin and $E_d = 30$ MeV for our 3-D Langevin calculation with macroscopic transport coefficients. In our 3D-Langevin calculations with microscopic transport coefficients, we use full shell corrections where $\Phi(E_{int}) = 1$. The intrinsic energy E_{int} is calculated at each time step with respect to excitation energy E_x ,

$$E_{int} = E_x - \frac{1}{2}m_{\mu\nu}^{-1}p_{\mu}p_{\nu} - U(q).$$
⁽⁵⁾

This bring us to the next essential ingredient to Langevin equation are the transport coefficients consisting of the inertia $m_{\mu\nu}^{-1}$ and friction $\gamma_{\mu\nu}$ tensors. Macroscopic transport coefficients are formulated based only on the shape of the nucleus but microscopic transport coefficients are calculated from microscopic approach.

2.1 Macroscopic transport coefficient

From the two definitions of kinetic energy,

$$T = \frac{1}{2}\rho_0 2\pi \int_{z_L}^{z_R} \rho^2 \, \vec{v}.\vec{v} \, dz = \frac{1}{2} \sum_{\mu\nu} M_{\mu\nu} \dot{q}_{\mu} \dot{q}_{\nu} \tag{6}$$

with the constant density defined by $\rho_0 = A/(4\pi R_0^3/3)$ and the nucleus assumed to be incompressible fluid, we can derive the Werner-Wheeler macroscopic mass tensor [19]

$$M_{\mu\nu}^{WW} = \pi \rho_0 \int_{z_L}^{z_R} \rho^2 \left[A_{\mu} A_{\nu} + \frac{\rho^2}{8} A_{\mu}^{'} A_{\nu}^{'} \right] dz, \tag{7}$$

with

$$A_{\mu}(z;q) = \frac{1}{\rho^2(z,q)} \frac{\partial}{\partial q_{\mu}} \int_{z}^{z_R} \rho^2(z',q) dz'.$$
(8)

On the other hand, the two relationship for the loss of collective energy according to Blocki [20],

$$\dot{E} = \frac{3}{4}\rho_0 v_f \oint_S v_n^2(s) ds = \sum_{\mu\nu} \gamma_{\mu\nu}^{\text{wall}} \dot{q}_\mu \dot{q}_\nu \tag{9}$$

could be used to obtain the wall friction $\gamma_{\mu\nu}$. v_n is the velocity relative to the nuclear surface and $v_F = \hbar k_F/m$ is the Fermi velocity. The Fermi momentum can be estimated from Fermi gas as $k_F R_0 = \sqrt[3]{9\pi A/4}$. If we wrote $v_n(s)$ in terms of $\rho(z;q)$, the wall friction could be written as

$$\gamma_{\mu\nu}^{\text{wall}} = \pi \rho_0 v_F \int_{z_L}^{z_R} dz \, \frac{\partial \rho^2}{\partial q_\mu} \frac{\partial \rho^2}{\partial q_\nu} \left[4\rho^2 + \left(\frac{\partial \rho^2}{\partial z}\right)^2 \right]_{.}^{-1/2} \tag{10}$$

As the fragments begin to separate, the wall friction is expressed as the sum of both left and right fragment [4,21],

$$\gamma_{\mu\nu}^{\text{wall2}} = \frac{\pi\rho_0 \bar{v}}{2} \left(\int_{z_L}^0 I_L(z) \, dz + \int_0^{z_R} I_R(z) dz \right),\tag{11}$$

with

$$I_{L,R}(z) = \left(\frac{\partial\rho^2}{\partial q_{\mu}} + \frac{\partial\rho^2}{\partial z}\frac{\partial z_{cm}(L,R)}{\partial q_{\mu}}\right) \left(\frac{\partial\rho^2}{\partial q_{\nu}} + \frac{\partial\rho^2}{\partial z}\frac{\partial z_{cm}(L,R)}{\partial q_{\nu}}\right) \left[4\rho^2 + \left(\frac{\partial\rho^2}{\partial z}\right)^2\right]_{-1/2}^{-1/2}$$
(12)

Swiatecki [22] furthered the argument that as the fragment became close to a scission configuration, a window of size $\Delta \sigma = \pi r_{neck}^2$ should be introduced to allow nucleons from each fragment to traverse to the other fragment. This *window* friction term is written as

$$\gamma_{\mu\nu}^{\text{window}} = \frac{\rho_0 \bar{v}}{2} \left[\Delta \sigma \frac{\partial R_{12}}{\partial q_\mu} \frac{\partial R_{12}}{\partial q_\nu} + \frac{32}{9\Delta \sigma} \frac{\partial V_L}{\partial q_\mu} \frac{\partial V_L}{\partial q_\nu} \right],\tag{13}$$

Thus at scission configuration, the amount of friction experienced should be equal to the sum of both friction term, hence the term *wall-window* friction tensor,

$$\gamma_{\mu\nu}^{w+w} = \gamma_{\mu\nu}^{\text{wall2}} + \gamma_{\mu\nu}^{\text{window}}.$$
(14)

Nix and Sierk [23] then suggest that as the nucleus separates, there should be smooth transition from $\gamma_{\mu\nu}^{wall}$ to $\gamma_{\mu\nu}^{w+w}$ governed by $\alpha = (r_{\text{neck}}/R_{\text{min}})^2$. Here R_{min} is the minimal semi-axes of two outer ellipsoids in three-quadratic-surfaces shape parameterization. This ansatz is written as

$$\gamma_{\mu\nu}^{\text{total}} = \sin^2(\pi\alpha/2)\gamma_{\mu\nu}^{\text{wall}} + \cos^2(\pi\alpha/2)\gamma_{\mu\nu}^{w+w},\tag{15}$$

Normally this friction is very large and it is common practice to multiply it by a factor 0.27 in most applications.

2.2 Microscopic transport coefficients

The formulation of microscopic transport coefficients on the other hand is done with the linear response approach to the nuclear collective motion [24]. In the current calculation, pairing effects can be included by using independent quasi-particles Hamiltonian as suggested in [25],

$$\hat{H}_{BCS} = \sum_{k} 2v_k^2(\varepsilon_k - \lambda) - 2\Delta \sum_{k} u_k v_k + \frac{\Delta^2}{G} + \sum_{k} E_k(\alpha_k^{\dagger} \alpha_k + \alpha_{\bar{k}}^{\dagger} \alpha_{\bar{k}}).$$
(16)

 λ is the chemical potential, G the pairing strength constant and Δ the pairing gap. The variables u_k and v_k are the Bogoliubov-Valatin transformation coefficient. $\alpha_{\bar{k}}^{\dagger}$ and $\alpha_{\bar{k}}$ are the creation and annihilation operators for the quasi particles and the quasi-particle energies itself, $E_k = \sqrt{(\varepsilon_k - \lambda)^2 + \Delta^2}$. The \hat{F} operator in quasi-particle representation has the form,

$$\hat{F} = \sum_{k} F_{kk} 2v_k^2 + \sum_{jk} F_{kj} \xi_{kj} \left(\alpha_k^{\dagger} \alpha_j + \alpha_{\bar{k}}^{\dagger} \alpha_{\bar{j}} \right) + \sum_{kj} F_{kj} \eta_{kj} \left(\alpha_k^{\dagger} \alpha_{\bar{j}}^{\dagger} + \alpha_{\bar{j}} \alpha_k \right).$$
(17)



(a) Mass Tensor

(b) Friction Tensor

Fig. 3: Comparison of microscopic and macroscopic transport coefficients varied along z_0/R_0 with $\alpha = 0.18472$ and $\delta = 0.2$ fixed.

Denoting $E_{kj}^- \equiv E_k - E_j$, $E_{kj}^+ \equiv E_k + E_j$, $n_k^T = 1/(1 + e^{E_k/T})$, $\xi_{kj} \equiv u_k u_j - v_k v_j$ and $\eta_{kj} \equiv u_k v_j + v_k u_j$, we can write the following response function,

$$= -\frac{2\Theta(t)}{\hbar} \sum_{kj}^{\prime} (n_k^T - n_j^T) \xi_{kj}^2 F_{\mu}^{jk} F_{\nu}^{kj} \sin(E_{kj}^- t/\hbar) -\frac{2\Theta(t)}{\hbar} \sum_{kj} (n_k^T + n_j^T - 1) \eta_{kj}^2 F_{\mu}^{jk} F_{\nu}^{kj} \sin(E_{kj}^+ t/\hbar),$$
(18)

The diagonal components of ' ξ '-term of operator \hat{F} commute with the Hamiltonian, hence $\tilde{\chi}_{\mu\nu}(t)$ does not commute with the Hamiltonian. So the first summation are marked with a ' and the second sum was not marked because both diagonal and non-diagonal terms contribute. The Fourier transform of the response function leads to

$$\chi_{\mu\nu}(\omega) = \sum_{jk}^{\prime} \frac{(n_k^T - n_j^T)\xi_{kj}^2}{\hbar\omega - E_{kj}^- + i\epsilon} F_{\mu}^{jk} F_{\nu}^{kj} + \sum_{jk} \frac{(n_k^T + n_j^T - 1)\eta_{kj}^2}{\hbar\omega - E_{kj}^+ + i\epsilon} F_{\mu}^{jk} F_{\nu}^{kj}.$$
 (19)

Thus we are able to write the mass and friction tensor that also took into account the effects of pairing,

$$\gamma_{\mu\nu}(0) = 2\hbar \sum_{jk}' (n_k^T - n_j^T) \xi_{kj}^2 \frac{E_{kj}^- \Gamma_{kj}}{\left[(E_{kj}^-)^2 + \Gamma_{kj}^2 \right]^2} F_{\mu}^{kj} F_{\nu}^{jk} + 2\hbar \sum_{jk} \left(n_k^T + n_j^T - 1 \right) \eta_{kj}^2 \frac{E_{kj}^+ \Gamma_{kj}}{\left[(E_{kj}^+)^2 + \Gamma_{kj}^2 \right]^2} F_{\mu}^{kj} F_{\nu}^{jk} , \qquad (20)$$
$$M_{\mu\nu}(0) = \hbar^2 \sum_{jk}' (n_k^T - n_j^T) \xi_{kj}^2 \frac{E_{kj}^{-2} [E_{kj}^- - 3\Gamma_{kj}]}{\left[(E_{kj}^-)^2 + \Gamma_{kj}^2 \right]^3} F_{\mu}^{kj} F_{\nu}^{jk}$$

$$+\hbar^{2} \sum_{jk} \left(n_{k}^{T} + n_{j}^{T} - 1 \right) \eta_{kj}^{2} \frac{E_{kj}^{+2} [E_{kj}^{+} - 3\Gamma_{kj}]}{[(E_{kj}^{+})^{2} + \Gamma_{kj}^{2}]^{3}} F_{\mu}^{kj} F_{\nu}^{jk} \,. \tag{21}$$

We see gradual changes from low to high temperature in microscopic transport coefficient in Fig. 3 and we see that it coincides closely with macroscopic transport coefficients at some temperature choice. The macroscopic mass tensor described in Eq. (7) coincides with microscopic mass tensor

at low temperature. Meanwhile, we can also see that the macroscopic mass tensor is clearly too large if it is not factored by 0.27 but the profile is similar.

2.3 Random force and other parameters

The final term of Langevin equation in (2) also includes a random force term, determined as the product of white noise R and the strength factors $g_{\mu\nu}$. The random force strength is related to diffusion,

$$D_{\mu\nu} = g_{\mu\sigma}g_{\nu\sigma}.\tag{22}$$

In our calculation, the diffusion tensor is calculated as the product of effective temperature T^* and friction tensor through the modified Einstein relation,

$$D_{\mu\nu} = T^* \gamma_{\mu\nu}.$$
 (23)

Messiah in Eq. (XII.52) of his famous textbook on Quantum Mechanics [26] introduced the energy of the harmonic oscillator and its relationship with temperature T. It was later re-introduced by H. Hofmann [27, 28] to show that the diffusion tensor should not approach zero at very low temperature and with respect to effective temperature at zero point energy $\hbar \varpi/2$ and temperature of the system T,

$$T^* = \frac{\hbar \varpi}{2} \coth\left(\frac{\hbar \varpi}{2T}\right),\tag{24}$$

At low temperature we will get the ground energy for the harmonic oscillator. This zero point energy [29, 30] is estimated to range between 0.45 MeV to 2.23 MeV but it is computationally expensive to exactly calculate the zero-point energy at each time step in our calculations, so we often use a constant $\hbar \varpi = 2$ MeV which corresponds to the zero-point energy of 1 MeV. As a consequence, diffusion depends only on friction temperature below 1 MeV.

Temperature itself is estimated in our calculations through Fermi gas relation,

$$E_{\rm int} = aT^2. \tag{25}$$

Here E_{int} is obtained from Eq. (5), and related to T through level density, a. In our calculations, the level density we use is [31], *i.e.*,

$$a = \frac{A}{14.61} \left(1.0 + \frac{3.114}{\sqrt[3]{A}} + \frac{5.626}{\sqrt[3]{A^2}} \right).$$
(26)

Admittedly, any of the choices $\hbar \varpi$ or *a* could prove essential in exact reproduction of experimental fission observables but we find it surprisingly good enough for studying the interplay of fission observables and further understanding of the fission process as we shall explore in the next section.

3. Summary of recent results

Considerable progress has been made in the Langevin calculation. In 3D-Langevin calculations, it used to be very difficult to study the fission fragment yield at very low excitation energy. With the introduction of effective temperature in the modified Einstein equation in Eqs. (43) and (44),



tron fission yield in open circle.

(a) 236 U calculation at $E_x = 20$ MeV in red (b) 258 Fm calculation at $E_x = 3.34$ MeV in line and JENDL data for 14 MeV incident neu- red line and experimental data from spontaneous fission yield [32] open circle.

Fig. 4: Fission fragment yield with microscopic transport coefficients and full shell corrections.

we have been able to obtain a reasonable fission fragment yield for comparison with fission due to incident thermal neutrons and compare them with both experimental and evaluated data. The reason our Langevin calculation had a difficulty to begin with was due to our practice of starting our calculation at the minimum potential whenever possible. Thus the trajectory of the Langevin equation had to cross the potential saddle. As it gets close to the saddle, Eq. (5) at such configurations becomes minute. Thus the temperature determined from Eq. (25) also becomes very small. As a result, there is hardly any diffusion across the potential saddle and even lesser chance for that particular event to achieve scission configurations.

Most investigators seem to sidestep this problem by starting their equations on top of the second saddle instead. This should be fine if the potential surface is well understood and all the possible saddle points that leads to scission are well accounted. In our opinion however, it is more natural if possible to allow the trajectories to search the saddle itself, and if the energy allows, cross the saddle to scission configuration. In our recent publications [7], we have produced a reasonably good fission fragment yield in comparison to evaluated results in Fig. 4a for ²³⁶U. In the case of our ²⁵⁸Fm for comparison with spontaneous fission fragment yield, it is simply impossible for the trajectories to cross the barrier given that E_x is much lower than the barrier that will lead to scission. As such, we had no choice but to allow the trajectories to begin at the second barrier with excitation energy of 1 MeV above the barrier, equivalent to $E_x = 3.34$ MeV. The result for this calculation with ²⁵⁸Fm is given in Fig. 4b and compared with experimental data. It must be noted that we were initially surprised that our fission yield was able to produce this single peak fission yield for ²⁵⁸Fm and that it came quite close to the results from D. C. Hoffman [32].

This emergence of the single peak fission yield as opposed to double peak fission fragment yield is somewhat mysterious in the past. Since our Langevin calculation can reproduce the fission fragment yield, it might be instructive to study how the trajectories evolve with respect to the potential energy surface, U. The trajectories for these two calculations can be examined in Fig. 5 plotted with respect to z_0/R_0 and δ . Additionally, the potential energy surface is minimized



Fig. 5: 3D-langevin with microscopic transport coefficient trajectories across potential energy surface (deformation energy) minimized with respect to α and plotted as a function of z_0/R_0 and δ .

with regards to mass-asymmetry α . We see that in this configuration, there is a huge barrier for configurations with negative δ and large elongation. This prevents very oblate fragment shapes from emerging. In the case of ²³⁶U, this barrier extends all the way to $\delta \approx 0$. Hence, almost all the trajectories end up with $\delta \approx 0$ and such fragments are somewhat spherical. In the case of ²⁵⁸Fm, this barrier extends only up to $\delta \approx -0.3$. As a consequence, the trajectories are able to achieve configurations with $\delta < 0$ and shorter z_0/R_0 when they finally scission. This configuration means that the fragments are very oblate in shape and the elongation is shorter.



and prescission kinetic energy. Red and blue lines are respectively due to microscopic calculation [7] and macroscopic calculation [33]. The closed and open square are experimental results from Dyachenko [34] and Duke [35].

Fig. 6: TKE dependence on excitation energy for ²³⁶U

This oblate shape and the shorter elongation mean that the Coulomb repulsion is also much stronger for ²⁵⁸Fm. In fact, Brosa [36] has demonstrated that such shapes are referred to as supershort fission modes. Further examination of this shape also reveals that more often than not, the fission fragments also have $\alpha \approx 0$ which will lead to fragments that have very similar fragment masses as predicted by Brosa for this fission mode. This shape is also apparently the most numerous in the fission of ²⁵⁸Fm, giving it single peak fission yield.

The spherical fragment shape we described earlier for ²³⁶U can be attributed to the standard fission modes and they usually have $|\alpha| > 0$. Thus they will produce a light and heavy fragment which manifested as a double peak fission yield when tallied.



Fig. 7: TKE profile for ²⁵⁸Fm.

The standard fission mode is also occasionally produced for the fission of ²⁵⁸Fm when we use microscopic transport coefficients but it is not enough to affect the fission yield. With macroscopic transport coefficients, no such fission events are noted so far for ²⁵⁸Fm indicating that it emerges only due to the use of microscopic transport coefficients. We also note that the trajectories for ²³⁶U occasionally elongate as far as $z_0/R_0 \approx 3$ with $\delta \approx 0.2$. This means that the fragments in this fission events have a prolate shape associ-

ated with Brosa's super-long fission modes. True enough as predicted by Brosa, this trajectory also possesses symmetric mass fragments. This prolate and elongated shape also means that the Coulomb repulsion between the fragments is weaker than both super-short and standard fission modes.

The total kinetic energy (TKE) in our calculation is due to two contributions. The main contributions are of course due to Coulomb repulsion. Additional contributions came from the kinetic energy of the elongations itself just before scission and is often called pre-scission kinetic energy. By taking a tally of the mass and TKE of the fragments for each fission events, we can obtain the profile for ²³⁶U in Fig. 6a and if we do the same for ²⁵⁸Fm, we have Fig. 7. As we have noted earlier, most ²³⁶U fission events are due to standard fission events that have an average kinetic energy of around 175 MeV. There are also some minor contributions from super-long fission modes are increasing. We have also noted that in the case of ²⁵⁸Fm, the super-short fission modes are most dominant and they have a kinetic energy around 230 MeV. The standard fission modes are around 195 MeV for ²⁵⁸Fm but the contributions are very small.

The main advantages that we see in the use of Langevin calculation with microscopic transport coefficients are the improvements towards the predictions of TKE with respect to increasing excitation energy. In Fig. 6b, we do not see any change in TKE as an excitation energy increases if macroscopic transport coefficients are used in 3D-Langevin calculation. On the other hand, with microscopic transport coefficients TKE not only decays with E_x as in experiment, it was also able to show the effects of pairing at very low excitation energy. We have early indication showing that further improvements may be achievable if better estimates of level density are available as was demonstrated in the work by [37]. The success of 3D Langevin microscopic calculation in predicting the TKE might be related to the dependence of microscopic transport coefficients on temperature and we are still investigating how this is actually manifested in our calculations.

4. Final words

It is interesting to see the influence of fission modes on TKE. In our recently published results [7] the average TKE of many nuclei seems to follow the Viola systematics pretty closely except for a few nuclei such as the spontaneous fission of ²⁵⁸Fm among others. The only differences that we can see between ²⁵⁸Fm and ²³⁶U that we can observe are the contributions of standard fission modes to the average TKE. It can be argued that the standard fission modes are prevalent in most nuclei, hence such systematics are a good measure on the systematics of standard fission modes. Due to the lack of these fission modes in ²⁵⁸Fm, its TKE appears to be anomalously high. In future work, we hope to study this subject further using the newly developed 4D Langevin calculation [8] to improve our understanding of the phenomena.

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