Nuclear Data for Science and Technology (1988 MITO), 1225-1230, Copyright © 1988 JAERI.

THEORETICAL PHOTOFISSION CROSS SECTIONS OF SOME ISOTOPES

L.S. El-Mekkawi, O.M.El-Bakly
Hot Laboratory Centre,Atomic Authority of Egypt.

Abstract: The present work considers that nuclei may contain a quantum system of two packed clusters. The deuteron which is considered as a two body problem was solved as the case of lightest clusters.

The photofission cross section has been estimated for deuteron in the energy range 2.35-450 Mev. The results of calculation give a curve with two peaks. One peak has a cross section 3.2 m. barn at energy 4.4. Mev. and another peak has a cross section 0.081 m. barn at energy 280 Mev.

The results agree well with experimental data.

For heavy fissionable nuclei 233-U, 235-U and 239-Pu, the disturbance of the system of two packed clusters may show a release of a saturated quantum shell. At a certain energy threshold gamma rays can resolve two equal clusters, if a quantum shell, with the higher quantum number of the bigger cluster, leaves cluster motion.

It was shown that photofission may originate from the substates of quantum system of clusters, when a ray resolves them. The number of final fission states is that of the order of thedisturbed shell, in case of main resolution of both clusters. But when the ray resolves the equal cluster substates, the number of final fission states is the number of substates of the disturbed shell.

The number of channels released from packed cluster system is in excellent agreement with number of fission product chains. Relation of thresholds allows energy of shell release. The two modes of fission give close agreement with experiments and explain the low energy bump in photofission cross sections.

Introduction

The present theory assumes that, photofission starts from two clusters within some nuclei, It is of particular interest to study the simplest nuclear and cluster system i.e.the deuteron which almost certainly consist of a proteon and a neutron. The work of Heisenberg Majorana /2/, Wigner /3/, Bethe and Peierl /4/ showed that protons and neutrons within the nucleus may be described by methods of quantum mechanics /5/6/

Our study of deuteron as a quasi-molecule shall be generalised by increasing the order of quantum system of clusters to solve the case of heavy nuclei.

The nulcear fission of heavy elements by e. m. energy was predicted by Bohr & wheeler./7/

The crossection for mono energetic -rays can be calculated by the "Photon difference" method described by Katz and Cameron /8,9/. Penfoldand Leiss developed an inverted matrix method for such computation /10/

Gold haber and Teller /11/, advanced an explanation of photofission based on vibration of the bulk of protons in a direction of motion opposite to neutrons at a fixed resonance frequency.

The present work hopes to explain photofssion cross section at low energies./11-20/ The three fissionable isotopes u-233, u-235, pu -239 were chosen because they are more abundant in literature, and have masses sufficiently different to allow compariosn between results.

Wave equation for two interacting subgroups influenced by radiation :

The Hamilitonian for two clusters with momentum p_1, P_2 & masses m_1, m_2 & interacting potential V(r) is

$$H= P_1^2/2m_1 + P_2^2/2m_2 + V(r) \qquad (1)$$

In presence of e.m.field, the Hamiltonian is

$$H=\frac{1}{2m_1}\left(P_1-\frac{z_1 e}{c} A_1\right)^2+(P_2- z_2 e A_2/c)^2/2m_2 + z_1 e \varphi_1 + z_2 e \varphi_2 + V(r) \qquad (2)$$

Where A and $\emptyset$ are the operators of Magnetic vector potentialand scaler potention of e.m.field

$$\bar{P}.\bar{A}(r)-\bar{A}(r)\bar{P} = -i\hbar\, \mathrm{div}\bar{A}(r) \qquad (3)$$

$$\mathrm{div}.A = 0;\ \emptyset = 0 = e^2A^2/2mc^2$$

$$H=P_1^2/2m_1+P_2^2/2m_2+V(r)- (e/c)\left[(z_1/m_1)\bar{A}_1.\bar{P}_1 +(z_2/m_2)\bar{A}_2.\bar{P}_2\right] \qquad (4)$$

In a nucleus some neutrons & protons may be United in certain structuralquanties subgroups, forming clusters.

We define a pair of packed cluster as belonging to one quantum system.
While each cluster can also have an individual quantum order.
From this definition the angular momentum of a packed cluster level , is the same for both clusters .

For packed cluster at different resolution the nucleons of one level have equal number of states S for different quantum order. Thus the number of states "S" per level per cluster is assumed equal for all quantum order see Fig. 1x. Thus nucleons may exchange amount equal order cluster levels making symetric force of binding inergy almost equal per nucleon in each sub cluster level or the charge to mass ratio is nearly equal for both sub clusters.

As one of the clusters have a higher order by one unit, this higher order level which does not exchange nucleons is assumed to be saturated with neutrons, for fissionable nuclei.

Also for fissionable nuclei, the average charge to mass ratio per cluster is higher than the rest of nucleons, which lowers the binding energy of the rest of nucleus.

Therefor each cluster can also astisfy the Hamiltonian where V(r) is interacting potential between the two clusters. Where $m_1 \doteq A_1 m_n$ of the first cluster.

$m_2 \doteq A_2 m_n$ for second cluster and z is cluster's charge number.

For these quantised clusters the moentum is given by $- i \hbar$. Transiforming to coordinates of center of mass and relative position r of clusters, where,

$$R = (m_1 r_1 + M_2 r_2)/(m_1 + m_2)$$

$$r = r_1 - r_2 \;;\mu = m_1 m_2/(m_1 + m_2) \qquad (6)$$

$$H = H_R + H' + H_o \qquad (7a)$$

The Hamiltonian of unperturbed system H_o is

$$H_o = \frac{-\hbar^2}{2\mu} \nabla_r^2 + V(r) \qquad (7b)$$

The Hamiltonian of centre of mass system H_R

$$H_R = \frac{-\hbar^2}{2(m_1+m_2)} \nabla_R^2 + (z_1\bar{A}_1 + z_2\bar{A}_2)(e/c(m_1+m_2))$$

$$(\hbar/i)\nabla_R \qquad (7c)$$

The perturbation Hamiltonian

$$H' = (e/c)\left[(z_1/m_1)\bar{A}_1 - (z_2/m_2)\bar{A}_2\right](\hbar/i)\nabla_r \quad ....(7d)$$

By putting $A_1 = A_2 = 0$ in H, Schrodinger equation is obtained

$$\left[(-\hbar^2/2\mu)\nabla_r^2 + V(r) - (\hbar^2/2(m_1+m_2))\nabla_R^2\right] \psi(R,r) = E\ \psi(R,r)$$

For rleative motion

$$\left[\nabla_r^2 + (2\mu/\hbar^2)\ (V(r)-E)\right] \psi(r) = 0 \qquad (7e)$$

Putting $V(r) = \frac{1}{2}\mu w^2 r^2$ (8)

$$\psi_s = N_l r^l e^{-\alpha r^2}\ Y_l^m(\theta',\phi') \qquad (9)$$

$$\varphi(\phi') = (1/2\pi)^{1/2} e^{im\phi'} \qquad (9a)$$

$$\theta_{lm}' = \frac{(-)^l}{2^l\, l!} \frac{(2l+1)}{2} \frac{(l-m)!}{(l+m)!} \sin^m\theta' \frac{\partial^{l+m}}{(\partial\cos\theta')^{l+m}} \sin^{2l}\theta \quad ......(9b)$$

$$Y_l^m = \varphi' \times \theta' \qquad (9c)$$

$$N_l = \left[2(\mu w/\hbar)^{3/2}/\Gamma(1+3/2)\right](\mu w/\hbar)^{l/2} \qquad (9d)$$

$$\alpha = \mu w/2\hbar \qquad (9e)$$

For ray propagation along $\phi = 0$, magnetic vector potential A along $\theta = 0$, the polarisation direction is inclined at (α, β) relative to ray system.

As $A_\phi = 0$ the initial solution responding to the photon is a plane solution part $P(\theta')$

$$P(\theta') = (4\pi/(2l+1))^{1/2}\left[\sum_{m=-l}^{m=l} Y_l^m(\alpha,\beta)\ Y_l^m(\theta,\varphi)\right] \quad ....(10)$$

$$\text{But}\quad Y_l^{m*} = (-)^m\, Y_l^{-m} \qquad (11)$$

$$\text{Putting}\quad Y_l^m(\theta,\phi) = N_l^m\, e^{im\phi}\, f_l^m(\theta) \qquad (12)$$

$$p(\theta') = (4\pi/(2l+1))^{1/2}\left[\sum_{m=1}^{m=1} 2(N_l^m)^2 \cos(\phi-\beta)m\right.$$

$$f_l^m(\alpha)\, f_l^m(\theta) + (N_l^o)^2\, f_l^o(\alpha)\, f_l^o(\theta) \quad ....(13)$$

Therefore the initial solution ψ_i effective to radiation field becomes

$$\psi_i = (4\pi/(2l+1))^{1/2} N_l r^l e^{-\alpha r^2}\ \psi_u \qquad (14)$$

where

$$\psi_u = \sum_{m=1}^{l} 2(N_l^m)^2 \cos m(\phi-\beta)\ f_l^m(\alpha) f_l^m(\theta) + (N_l^o)^2 f_l^{o}(\alpha)\, f_l^o(\theta) \qquad (15)$$

Absorbtion of -rays

The perturbation Hamiltonian for the ray equivalent magnitic vector potential A at position of charged clusters as from eq.(1)

$$H' = (z_1/m_1)e^{ikr_1} - (z_2/m_2)e^{ikr_2}$$

$$(e/c)\ \frac{A.\hbar}{i}\ \bar{\nabla}_r \qquad (16)$$

The matrix defining perturbation is

$$H_{ki} = \int \psi_k^*\, H'\, \psi_i\, d\tau \qquad (17)$$

Where ψ_k is the normalised final function for free motion.

$$\psi_k = k^{3/2} \sum (i)^{l_f}\, Y_{l_f}^m(\hat{k})\, Y_l^m(\hat{r}) \times J_{l_f}(\rho)/\rho^{1/2}$$

$$= k^{3/2}\left[J(\rho)\ /\rho^{1/2}\ F(\theta,\phi)\right. \qquad (18)$$

Where $\rho = kr$ & $\hat{K}$ has the direction $(\frac{\pi}{2}, 0)$ and $\hat{r}$ has the direction (θ, ϕ).

on substtituting the value of the initial solution relative to radiation field ϕ_i by eq (14)

$$H_{ki} = (4\pi/(2l+1))^{1/2} \frac{e\hbar}{c} N_l\, A_o \int r^{l-1} e^{-\alpha r^2}/_k +$$

$$\left|(z_1/m_1)e^{ikr_1}-(z_2/m_2)e^{-ikr_2}\right|(\ell\psi_u\cos\theta-\sin\theta\frac{\partial\psi_u}{\partial\theta})-2\alpha r^2\cos\theta \quad d\tau \qquad (19)$$

Where r_1 and r_2 are relative initial positions of the first and second cluster at moment of ray interaction.

$$H_{ki}=(e/c)\hbar A_o\left|(z_1/m_1)e^{ikr_1}-(z_2/m_2)e^{-ikr_2}\right|J'_H$$

$$J'_H=(4\pi/(2\ell+1))^{1/2}N_\ell\int\psi_k^* r^{\ell-1}e^{-\alpha r^2}[(\ell\psi_u\cos\theta-\sin\theta\,\partial\psi_u/\partial\theta)-2\alpha r^2\cos\theta]\,r^2dr\,d\Omega \quad ....(20)$$

It is possible to separate the integral J_H' into a radial part Ir,1 and a spherical Harmonic part J_H

$$I_{r,\ell}=N_\ell\int r^{\ell+1/2}e^{-\alpha r^2}\,J_{\ell_f+1/2}(kr)\,dr \qquad (21)$$

Where N_1 is the radial normalisation constant of the initial function

$$(N_1)^2\int r^{2l}e^{-2\alpha r^2}r^2\,dr=1$$

$$N_\ell=(2\alpha)^{(\ell+3/2)/2}N'_\ell \qquad (14a)$$

where N_ℓ' is dimensionaless.

As we assume.

$$\ell_f=\ell-1=\vartheta-1/2 \qquad (22)$$

and the propoagation energy of both fragments

$$\hbar^2k^2/2\mu'=\hbar\omega-E_{th} \qquad (24)$$

$$\text{putting } u=k^2/2\alpha \qquad (23)$$

E_{th} is cluster binding energy

ω is incident radiation frquency
K is propagation constant for final function
μ' is reduced mass of both clusters at final frunction state.

$$k^2/2\alpha=2\mu'E_{th}(h\omega/E_{th}-1)=u \qquad (24a)$$

$$u=b(\gamma-1) \qquad (23a)$$

$$I_{r,\ell}=N'_\ell\,u^{\vartheta/2}\,e^{-u/2} \qquad (21a)$$

$$I_{r+2,\ell}=N'_\ell\,u^{\vartheta/2}(2\ell+1-u)e^{-u/2} \qquad (25)$$

From & eq (20,24) and eq(25)

$$J'_H=(4\pi/(2\ell+1))^{1/2}u^{\vartheta/2}e^{-u/2}N'_{1k}\int F(\theta,\phi)\,[\ell\psi_u\cos\theta-\sin\theta\,\partial\psi_u/\partial\theta-((2\ell+1)-u)\psi_u\cos\theta]\,d\Omega \qquad (26)$$

As F (θ,ϕ) and ψ are spherical harmonic functions the intégration canbe obtained for values of quantum number ℓ.

Time phases of perturbantion:

The ray has an equivalent time varying amplitude of magnetic vector potential. It also causes transition.of quantum states defined by perturbatiuon Hamiltonian. Thus the time coefficient C(t) is

$$C(t)=-i\int H_{ik}e^{i(\omega_{rs}-\omega)t}\,dt/\hbar \qquad (27)$$

As the higher order shell contains neutrons only, it does not respond to ray angular momentum and may separate from cluster levels.

As it has the same rotation as the other clusters , its states give the number of nodes for final function motion normal to rotation.

These final function nodes are radially distributed for a solid angle element.

$$\rho(E)\,dE=n^2\,dn\,d\Omega \qquad (28)$$

$$\rho(E)=\frac{\mu' n^3}{\hbar^2k^2}\,d\Omega$$

or

The probability per unit time W

$$W=\int(4/t)\;H^*_{ik}\,\rho(E)\,\frac{\sin^2\frac{1}{2}(\omega_{ki}-\omega)}{\hbar^2\,(\omega_{ks}-\omega)}\times dE_{sk}$$

$$=2\pi\frac{\mu'\,n^3\,e^2\,A_o^2}{h\quad m_nc^2}\left|(z_1/m_1)e^{ikr_1}-(z_2/m_2)e^{-ikr_2}\right|u^{\vartheta}e^{-u}J_H^*\,J_H\;d\Omega \qquad (29)$$

As for e.m. wave the number of events per area perunit time $\dot{n}$ is

$$\dot{n}=\omega^2\,A_o^2\,/\hbar\omega\,(2\pi c) \qquad (30)$$

Therefore the crossection for gamma photofission is $w/\dot{n}$.

The samplest case of packed cluster is the deutron ., for which one nucleon occupies packed cluster order 1 and the second occupies cluster order 2 and the total pack cluster has order 3.

Thus the top quantum number for initial function is L = 3 giving substate 2 for final function & line L = 1 can also produece fission.

For packed cluster of quantum order L where are 1 equal substates for initial function.

Substituting the order and states in harmonic integral as defined by)29), (26) and (30) and noting that $m_1 = z_1 = 0 \& Z_2 0 = 0$ and that final state quantum number in(28) is = 1.

$$\sigma = \frac{(4\pi)^2}{2x\ 0.5!} (e\hbar/m_n c) \mu\, n^3 u^{1/2} \frac{e^{-u}}{E} \left[\frac{(1-u/3)^2}{1.5} + .01924198\ u^4\right]$$

with estimated value of reduced mass μ which is very nearly eqaal to halt.

$$\sigma = 135.77052 \frac{u^{1/2} e^{-u}}{E} \left[(1-u/3)^2/1.5 + 0.01924198\ u^4\right] \qquad (31)$$

The threshold value, ϵ_{th} ²H2 determined by chadwick & Goldhaber was found 2.1 x 10^6 volts. The value ϵ_{th} used in this work = 2.35 x 10^6 volt & the value of b for q (23a) is chosen be= 020234608 which gives a rotation energy E_i>155,63 Mev and explains why there is a min of E = 150 Mev. with 0 = 0.65 x m.b.

obtained from EQ (31)

Corresponding to expreimental value σ=,055 mb. at about the same energy. The second high energy peak (near double E_i) at 280 mev. originate from L = 3 has σ = ,091 mb . corresponding to σ = .065 m.b. experimental . The sublevel L=.1 gives the peak at 4.4 Mev with σ = 3.2 m.b. with expermental value 2.9 m.b. at the same energy. The experimental & the theoretical curves drop with the same slope at high energy end. as in fig.1.

For heavier Elements, the subclusters order is increased by one and therefore the packed cluster system is increased by 2 to get the order 5.

If the higher order cluster releases a quantum shell, the quatum number for final function is lowered by one unit.

The shell released has a split cluster order 3 and 18 internal states, (saturated).

The number of final fission states is that of the order to the disturbed shell in case of main resolution of packed cluster system. But when the ray resolves the equal cluster sub states, the number of final fission states, is the number of substates of the distrurbed shell.

As a packed cluster with five equal initial levels ends in 18 separate nuclear states, there are 5 x 18 = 90 fission. Channels. At final function 4 equal levels with 18 nucleon states, give a minimum final fission mass Of 4 x 18 = 72 mass unit.

Thus for 90 mass steps, the heaviest fission mass = 72 + 89 = 161 mass unit, which are exact values of tission data.

For packed cluster resolution at quantum order 5 equ (29, 30) give.

$$\sigma_5 = 271.779\ (2z_1/m_1)^2 \left[\frac{(\delta/2)^2}{1+\delta} + \sin^2(\pi E/2E_h)\right] \mu' n^3 u^{1/2} e^{-u} \left[(1-u/3)^2/1.5 + u^2\{4.6285(1-2u/21)^2 + 0.42857\} + 0.174191\ u^4(1-3u/26)^2 + .001595\ u^6\right] \qquad \ldots\ldots\ldots\ldots(32)$$

At sub level resolution by quantum order 2

$$\sigma_2 = 1.087265\ \mu'\ n^3 (2z_1/m_1)^2 \left[\frac{(\delta/2)^2}{1+\delta} + \sin^2(\pi E/E_h\ x2)\right] \frac{u^{3.5} e^{-u}}{E} \qquad \ldots\ldots (33)$$

For (Pu-239),(u-233), the original subclusters have relatively high charge causing first, higher photo fission. cross section and second higher symtric binding energy of the neutron shell of order 3, which prevents release of this shell at ray resolution of order 3.

For Pu - 239 the lower order cluster has 23 charge unit & 36 mass unit & the higher order cluster has 36 charge unit & 54 mass unit, for which δ = .0416. The threshold for fission due to resolution of order 2 is 5 Mev. & the value of b -is taken 3.18. Fission from resolution of packed cluster starts at 9 Mev. & the corresponding value of be is taken 5.44. The calculated curve is close to experimental Fig. 2.

For (U-233), the charge of the lower order cluster has 23 units and its mass is 36 units. The higher order clusterhas 33 charge unit and 54 mass units. The deviation of charge to mass ratio is 0.045. Fission from resolution of packed cluster starts at 10 Mev & the value of b is taken 7.606 as in Fig. 3

For (U-235), the lower order cluster has 19 charge unit and 36 mass unit. The higher order cluster has 28 charge unit and 54 mass units. Fission starts for order 2 before & Mev, with b = 3. . At 8,7685 Mev. the shell of order 3 is separately resolved and the neutron shell is released, the remaining clusters get charge number 23 and 24 and mass number 36. The deviation in charge to mass ratio is 0,0416 and the value of b = 12.1235. This explains the dip in order 2 fission at 9 Mev. Fission from resolution of packed cluster starts at 11.5 Mev and the value of b is taken 10.5

The curve explains the low energy bump as in Fig. 4.

References:

1. W.Heisenberg, Zeitschrift Physik, (1932) 1, Vol. 78 (1932) 156+ vol. 80 (1933) 587.
2. E. Majorana, Zeitschrift fur Physik, Vol.82 (1932) 137.
3. E.Wigner, Zeitschrift fur Physik, vol 83 (1933) 253, Phys. Rev.Vo.43 (1933) 252.
4. H.Bethe and R.Peierls, Proc. Roy. Soc. A 148 (1935) 146.
5. Chadwick and M.Goldhaber, Nature, vol.134 (1934) 237.
6. J.S.Levinger, Phys.Rev. 97 (1955) 970.
7. N.Bohr and J.A.Wheeler, Phy.Rev.56(1939) 426.
8. G.C.Baldwin and Klaiber, Phys.Rev.'1(1947)3.
9. L.Katz and G.W.Camoron, Can.J.Phys. 29 (1951) 518.
10. A.S.Penfold and J.E. Leiss, Phys. Rev.114 (1959) 1332
11. M.Goldhaber andE.Teller, Phys.Rev. 74(1948) 1046.
12. L.D. Landau, E.M.Lifshitz, The Classical theory of field (4th edition) Pergmaon press, oxford. 1975.
13. H.Goldstien, Class. Mec. REading Mass Addison-Wesby Publishing Co., Inc., 1950 Section 7-3.
14. H.Eyring, The late John Wilter and G.E. Kimall Quntum chemistry, New York, John Wiley and Sons, Inc.,1958- P.46 and P.57.
15. Peter fon "Elementary Quantum Mechanics Addison Wesley Publishing Company 1968 P.190.
16. John L.Powell and Bernd Crasemann Quantum Mechanics. Addison-Wesley Publishing Co., Inc. 1961, Problem 7-9 P.231.

Aknowledgment:

Thanks are expressed to Prof. Dr. M. El-Nadi, for ideas on implications and systematic working of quasi - molecule.

For his intrest we express our deepest qppreciation.

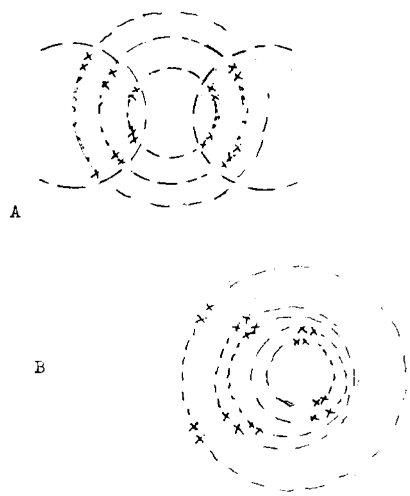


FIG 1x Resolution of packed cluster by order 2 in A and by order 5 in B.

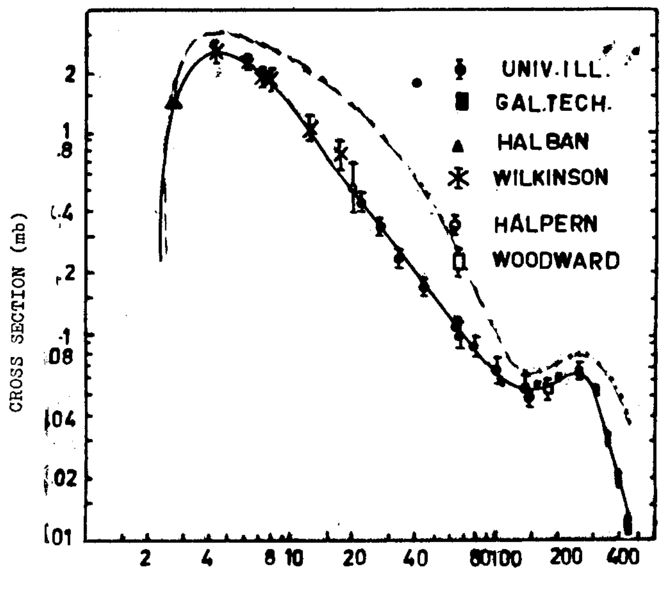


FIG.1 Deutron photofission cross section. Dashed line is theoretical cluster model value.

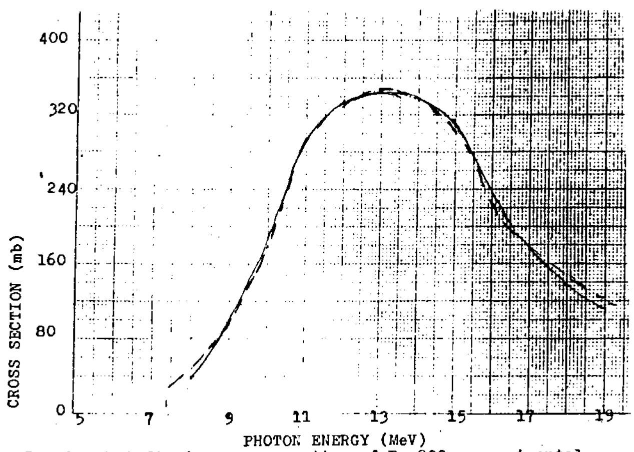
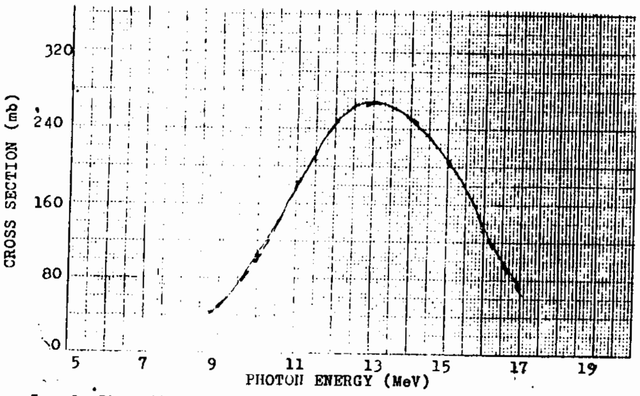


FIG.2. Photofission cross section of Pu-239 ,experimental in solid line, theoretical by packed cluster model in dashed line.



F... 3. Photofission cross section of U-233, experimental in solid line, theoretical by packed cluster model in dashed line.

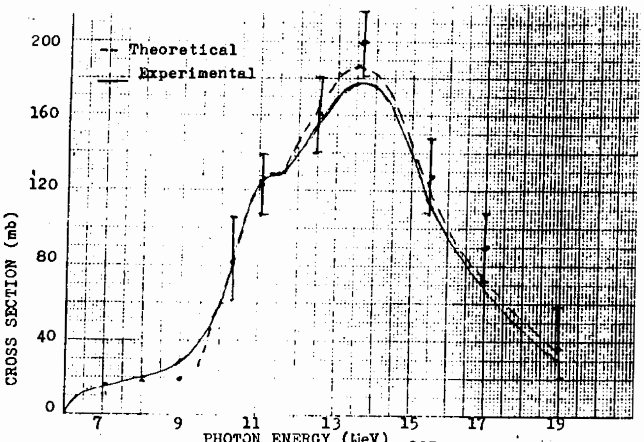


FIG.4. Photofission cross section of U^{235} measured by Bowman, Auchampaugh and Fultz. Dots are data from annihilation ofp positrons. Theoretical by packed cluster model.