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ENERGY AVERAGED RESONANCE NEUTRON CROSS SECTIONS

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Abstract: The basic assumptions of the statistical model are well determined, but there exist some difficulties in the practical calculations of the resonance cross sections. An analytical method of resonance cross sections averaging with account of interference and level width fluctuations at an arbitrary ratio level width to mean spacing has been developed. The expression for radiative capture cross section was obtained under certain approximations as a twofold integral which has to be calculated numerically. The appropriate computer COD was prepared. The case when radiative capture is the unique concurrent of the elastic scattering has been investigated and the values obtained for radiative cross section are in a good agreement with the Monte Carlo results. Further improvements of this method consider the two- and more channels case and the practical scheme for calculation of the transmission $\exp(-n\sigma)$ and other cross section functionals as group constants for reactor calculations. The careful analysis demonstrates the advantages of this method in precision and conversion.

(cross section, neutron resonance, level interference, analytical method)

The data for averaged values of transmission $\langle\exp(-n\sigma)\rangle$ and selfindication $\langle\sigma_\gamma\exp(-n\sigma)\rangle$ cross sections on energy intervals (groups) in dependence of the sample thickness n are of great importance for the resonance neutron transport problems. From theoretical point of view, the model of resonance structure containing all basic peculiarities of energy dependence essential for the transmission is needed for the determination of these values, especially in an unresolved resonance region. The correct description is important not only for the resonance maxima, but also for the interference minima. Evidently, this is possible only in the frame of multilevel approach.

The model proposed here uses a simplified version of R-matrix formalism. At present, it is limited only by the case of one level scattering in competition with the multilevel radiative capture (nonfissile nuclei). The energy structure of the cross sections in the case is determined by the function /1/

$$R(E)=\frac{1}{2}\sum_{\lambda}\Gamma_{\lambda n}/(E_\lambda-E-i\Gamma_\gamma/2) \qquad (1)$$

where $\Gamma_{\lambda n}$ are the neutron widths, E_λ - energies of resonances λ (Γ_γ - radiative width to be chosen equal for all λ). $\Gamma_{\lambda n}$, E_λ, Γ_γ are parameters in such a scheme.

The collision function S(E) and the cross sections $\sigma(E)$ and $\sigma_\gamma(E)$ are expressed through R(E) as /1-3/:

$$S(E)=e^{-2i\varphi}\left[\frac{2}{1-iR(E)}-1\right], \qquad (2)$$

$$\sigma = 2\,(1-\mathrm{Re}\,S) = 4\left[\cos^2\varphi - \mathrm{Re}\,e^{-2i\varphi}\,\frac{1}{1-iR}\right] \tag{3}$$

$$\sigma_\gamma = 1 - |S|^2 = 2i\,(R^* - R)\,/\,|1-iR|^2 \tag{4}$$

For determination of the average (for many resonances in the group) values of cross sections functionals $\langle F(\sigma)\rangle$ or $\langle F(R)\rangle$, the well known statistical distributions of resonance parameters in $R(E)$ (eq.1) are to be used. In the present work we suppose that the level spectrum is equidistant, namely - $E_\lambda = E_0 + \lambda D$, where D is the mean level spacing. The possible effect from the level spacing fluctuation in $\langle F(R)\rangle$ is taken into account by redetermination of parameter Γ_γ in eq.(1) as $\Gamma_\gamma \to \Gamma_\gamma + \Delta$ where Δ is the dispersion of $(E_\lambda - E_\lambda - 1)$ values distribution /4-5/. Then R(E) can be written as /3/:

$$R(E) = S\sum_{\lambda=-\infty}^{\infty} \frac{x_\lambda}{\mathcal{E} + \pi\lambda - iy} = R_1 + iR_2 \tag{5}$$

where $S = \pi\bar{\Gamma}_n/2D$, $y = \pi\Gamma_\gamma/2D$, $\mathcal{E} = \pi(E-E_0)/D$ and the values $x_\lambda = \Gamma_{\lambda n}/\Gamma_n$ are distributed as Porter-Thomas statistical law:

$$P(x)\,dx = \frac{1}{\sqrt{2\pi x}}\,e^{-x/2}\,dx \tag{6}$$

The average for a group containing a great number of resonances is determined as

$$\langle F(R)\rangle = \frac{1}{\pi}\int d\mathcal{E} \prod_\lambda \int_0^\infty F\Big(S\sum_\lambda \frac{x_\lambda}{\mathcal{E}+\pi\lambda - iy}\Big)\,P(x_\lambda)\,dx_\lambda \tag{7}$$

For many functionals F(R) the many-fold integral (7) can be analytically calculated /3/

$$\langle e^{iRt}\rangle = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\mathcal{E}\,\sqrt{\frac{\sin(\mathcal{E}-iy)}{\sin(\mathcal{E}-iy-2iSt)}}\;e^{-St} \tag{8}$$

$$\left\langle\left(\frac{1}{1-iR}\right)^k\right\rangle = \left(\frac{1}{1+S}\right)^k \tag{9}$$

$$\langle\sigma\rangle = 4\left[\sin^2\varphi + \frac{S}{1+S}\cos 2\varphi\right] \tag{10}$$

A more complicated expression for the many-fold integrals (7) is obtained in the case of functionals depending on both R and R^*, for example $\langle\sigma_\gamma\rangle$(4).

We determined the functional:

$$F = \langle e^{i(Rt - R^*t')}\rangle = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} d\mathcal{E}\left[\frac{\sin(\mathcal{E}-iy)\sin(\mathcal{E}+iy)}{\sin(\mathcal{E}-ip)\sin(\mathcal{E}+iq)}\right] \tag{11}$$

$$p = \zeta + s(t-t')$$

$$q = \zeta - s(t-t') \tag{12}$$

$$\zeta^2 = s^2(t+t')^2 + 2sy(t-t') + y^2$$

for which the presentation through full elliptic integrals exists of III order /5/.

The average radiative capture cross section (4) can be written as:

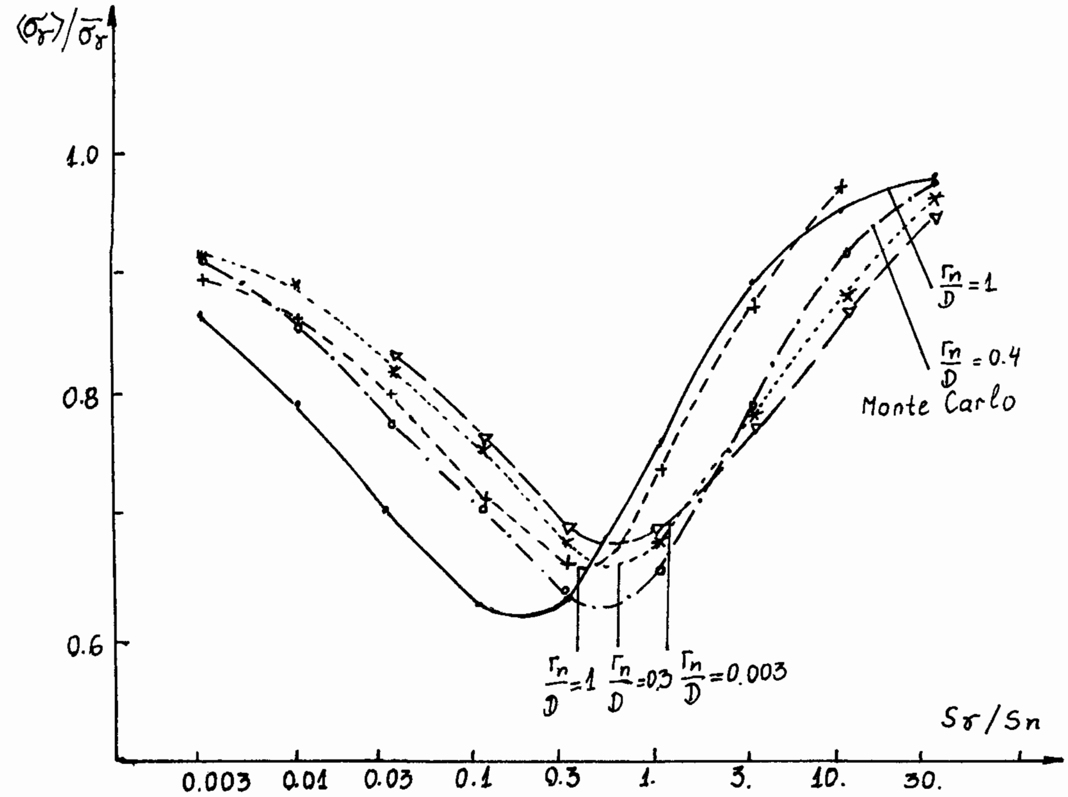
$$\langle\sigma_\gamma\rangle = -2\int_0^\infty\int_0^\infty e^{-(t+t')}\,dt\,dt'\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial t'}\right)F(t,t') \tag{13}$$

The derivative is expressed through an elliptic integral of II order

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial t'}\right)F(t,t') = \frac{4sy}{\pi\zeta}\,\frac{\sqrt{\mathrm{sh}(p+y)\,\mathrm{sh}(q+y)}}{\mathrm{sh}\,2\zeta}\,(1+k)\,E\!\left(\frac{2\sqrt{k}}{1+k}\right) \tag{14}$$

where

$$\kappa = \sqrt{\frac{sh(p-q)\, sh(q-y)}{sh(p+q)\, sh(q+y)}} \qquad (15)$$



$\langle\sigma_\gamma\rangle/\bar{\sigma}_\gamma$ in dependence of S_γ/Sn for different values of Γ_n/D

The ratio $\langle\sigma_\gamma\rangle/\bar{\sigma}_\gamma$, where $\bar{\sigma}_\gamma$ is the average cross section, without taking into account the widths fluctuations (the result with $x_\alpha = 1$ for R(E) (eq.5) /2/), is presented on the figure. The results of calculation of $\langle\sigma_\gamma\rangle$ in the frame of our model (13) are compared with the same value obtained by the Monte-Carlo method /3/. The figure shows good agreement between values obtained by both methods. The careful analysis demonstrates the advantages of this method in precision andconversion which is confirmed by numerical computer calculations.

Further improvements of this method consider the two and more channels case and the practical scheme for calculation of the transmission exp(-n) and other cross section functionals as group constants for reactor design.

The transmission averaged on energy group can be presented as

$$T = \langle \exp(-n\sigma)\rangle = e^{-A}\left\langle \exp\left(\frac{B}{g}+\frac{B^*}{g^*}\right)\right\rangle \qquad (16)$$

where $A = 4n\cos^2\varphi$, $B = 2ne^{-2i\varphi}$, $g = (1 - iR)$ (eq. 3).

Using the Laplace transformation /5/

$$e^{B/g} = g\int_0^\infty I_0(2\sqrt{Bt})\, e^{-gt}\, dt \qquad (17)$$

one obtains

$$T = e^{A}\int_0^\infty\int_0^\infty dt\, dt'\, I_0(2\sqrt{Bt})\, I_0(2\sqrt{B^* t^*}) \times P(t,t') \qquad (18)$$

where

$$P(t,t') = \langle g g^* e^{-gt} e^{-g^* t'} \rangle =$$
$$e^{-(t+t')} \left[\frac{\partial^2 F}{\partial t \partial t'} - \frac{\partial F}{\partial t} - \frac{\partial F}{\partial t'} + F \right] . \tag{19}$$

For the selfindication cross section in our model, the following expression can be obtained (3) (4):

$$\langle \sigma_\gamma \exp(-n\sigma) \rangle =$$
$$2 e^{-A} \int_0^\infty \int_0^\infty dt\, dt'\, e^{-(t+t')} I_0 \left(2\sqrt{Bt}\right) \times \tag{20}$$
$$\times I_0 \left(2\sqrt{B^* t'}\right)$$

where the derivative is determined by the relation (14).

<u>REFERENCES</u>

1. A. Lane, R. Thomas, Rev.Mod.Phys., 30, 1958, pp. 257
2. A.A. Lukyanov, Structure of neutron cross sections, Moscow, Atomizdat, 1978.
3. N. Yaneva et al. In Nucl. Data for Basic and Applied Science, Pros.Conf. Santa Fe 1985, p. 1615.
4. F. Fröhner. Ibid 3 p. 1541.
5. M. Abramowitz, I. Stegun, ed. Handbook of Mathematical Functions, 1964.