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SYSTEMATICS OF (N,2N) AND (N,3N) CROSS SECTIONS*

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Abstract: The systematics of (n,2n) and (n,3n) cross sections has been studied by means of the evaporation model including the preequilibrium emission. A set of the empirical parameters have been extracted on the bases of analysing and fitting the measured data. The (n,2n) and (n,3n) cross sections of some nuclei have been predicted, and good agreement with the measured data have been obtained.

(systematics, evaporation, preequilibrium, (n,2n) and (n,3n) cross sections)

Introduction

Nuclear data of (n,2n), (n,3n) reactions are of very importance for the designs of the fission and fusion reactor. Unfortunately, the measured data for some nuclei are very deficient because of the restriction of experimental conditions and technologies. The theoretical calculation depends closely upon nuclear model, but systematics require much less nuclear information and much less computational time than theory calculation. Therefore, the systematic study provides a reliable, simple and convenient method for predicting a certain cross section at a sufficient level of accuracy within a minimum amount of time.

Up to now, these works almost centred around 14 MeV, for $E_n \lesssim 14$ MeV energy range evaporation model is suitable[1,2]. However, with the incident neutron energy increase the competition of the (n,3n) reaction and contribution of preequilibrium emission can not be ignored.

In the present work, the systematic behaviour of (n,2n) and (n,3n) cross sections have been studied by using the evaporation model including preequilibrium emission developed by Segev et al[3]. The desired results also have been obtained.

Reduced Cross Sections Rmn(E)

We define a reduced cross sections Rmn(E) for neutron production reactions:

$$\sigma_{n,mn}(E)=\sigma_{n,M}(E)\cdot R_{mn}(E) \quad (m=1,2,3) \quad (1)$$

$\sigma_{n,M}$ is defined as the cross section for processes where only neutrons are emitted. $\sigma_{n,M}=\sigma_{n,n'}+\sigma_{n,2n}+\sigma_{n,3n}+\ldots$

In evaporation model, because medium and heavy mass nuclei have larger neutron excess and higher coulomb barrier than lighter, the production cross sections of charged-particle reactions, such as (n,p), (n,α) etc. are negligible. Thus for the reactions induced by fast neutron and emitting the secondary neutrons are main process. Then when the process achieves equilibrium(EQ), the general expressions of R2n(E) and R3n(E) may be

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written respectively as

$$R_{2n}^{EQ}(E)=\int_{W_3}^{W_2}P_1^{EQ}(W_1,E\to E_1)G_2(E,E_1)dE_1+\int_0^{W_3}P_1^{EQ}(W_1,E\to E_1)G_2(E,E_1)\times\left[\int_{W_3-E_1}^{W_3}P_2^{EQ}(W_2,E\to E_2)G_3(E,E_1,E_2)dE_2\right]dE_1 \quad (E<B_3) \quad (2.1)$$

$$R_{3n}^{EQ}(E)=\int_0^{W_3}P_1^{EQ}(W_1,E\to E_1)G_2(E,E_1)\times\left[\int_0^{W_3-E_1}P_2^{EQ}(W_2,E\to E_2)G_3(E,E_1,E_2)dE_2\right]dE_1 \quad (E<B_3) \quad (2.2)$$

where E is the incident neutron energy, Em is the mth outgoing neutron energy, Wm is the excitation energy of nucleus . Gm is the probability of emitting the mth neutron, and B_3 is the separation energy of three neutrons from nucleus A. It is assumed that neutron emission prevails over other modes of energy release whenever energy constraints don't prohibit neutron emission. In other words, approximately taking $G_2=G_3=1$.

By using the formulae of level density and normalized conditions and making some approximations we can get the expressions for the energy distribution of the mth outgoing neutron.

$$P_1^{EQ}(W_1,E\to E_1)\simeq\frac{1}{\theta_1^2}\cdot E_1\exp\left(-\frac{E_1}{\theta_1}\right) \quad (3.1)$$

$$P_2^{EQ}(W_2,E\to E_2)\simeq\frac{1}{\theta_2^2}\cdot E_2\exp\left(-\frac{E_2}{\theta_2}\right) \quad (3.2)$$

where

$$\theta_1=\left(\frac{E}{a_A}\right)^{\frac{1}{2}},\quad \theta_2=\left(\frac{E-B_1-2\theta_1}{a_{A-1}}\right)^{\frac{1}{2}} \quad (4)$$

where a is the characteristic parameter of the level density formula and B_1 is the separation energy of one neutron from nucleus A.

Substituting eqs. (3) in (2) and performing the integration yield the analytic expressions of $R_{2n}^{EQ}(E)$ and $R_{3n}^{EQ}(E)$.

As the increasing of incident neutron energy, the competition from the (n,3n) reaction and contribution of preequilibrium emission must be taken into accout in above formulae. Generally, assuming that preequilibrium emission can only occurs during the first outgoing neutron process. The others can be entirely attributed to the evaporation process. Similarly, the reduced cross sections of preequilibrium emission (PE) may be written respectively as

$$R_n^{PE}(E)=\int_{W_2}^{E}P_1^{PE}(W_1,E\to E_1)dE_1 \quad (5.1)$$

$$R_{3n}^{PE}(E)=\int_0^{W_3}P_1^{PE}(W_1,E\to E_1)\times\left[\int_0^{W_3-E_1}P_2^{EQ}(W_2,E\to E_2)dE_2\right]dE_1 \quad (5.2)$$

$$R_{2n}^{PE}(E)=1-\left[R_n^{PE}(E)+R_{3n}^{PE}(E)\right] \quad (5.3)$$

The energy distribution of the 1st outgoing neutron may be derived from exciton model.

$$P_1^{PE}(E\to E_1)=E_1\cdot\frac{E+B_1}{(E-E_1)^2}\cdot\sum_{n\geq 3}^{\bar{n}}\left(\frac{E-E_1}{E+B_1}\right)^n(n^2-n) \quad (6)$$

Generally, taking average exciton number $\bar{n}=3$. substituting eq. (6) in (5) and performing the integration yield the analytic expressions of $R_{2n}^{PE}(E)$ and $R_{3n}^{PE}(E)$.

Formulae for Data Fitting

The total neutron cross sections should be calculated as the sum of two kinds of the reaction mechanism on the bases of evaporation and preequilibrium model.

Letting δ to denote the fraction of preequilibrium emission in nonelastic cross sections, from eq. (1), we have

$$\sigma_{n,2n}(E)=\sigma_{ne}\cdot\frac{\sigma_{n,M}}{\sigma_{ne}}\cdot\left[(1-\delta)R_{2n}^{EQ}(E)+\delta R_{2n}^{PE}(E)\right] \quad (7.1)$$

$$\sigma_{n,3n}(E)=\sigma_{ne}\cdot\frac{\sigma_{n,M}}{\sigma_{ne}}\cdot\left[(1-\delta)R_{3n}^{EQ}(E)+\delta R_{3n}^{PE}(E)\right] \quad (7.2)$$

where $R_{2n}^{EQ}(E)$, $R_{3n}^{EQ}(E)$ and $R_{2n}^{PE}(E)$, $R_{3n}^{PE}(E)$ have been given in the eqs. (2) and (5).

We use a empirical formula[4] to compute the non-elastic cross sections σ_{ne}, while the normalized factor is of a well known form

$$\frac{\sigma_{n,M}}{\sigma_{ne}}=1-k\exp\left[-\frac{m(N-Z)}{A}\right] \quad (8)$$

Following the formulae of exciton model and taking some approximations we can get a simplified expression of the fraction δ of preequilibrium emission as follows[5]

$$\delta = 1-\exp\left\{-G\left[A(x)+A(y)\right]\right\} \quad (9)$$

where A(x) and A(y) are a functions of N, Z, E, and B_0 (neutron separation energy of nucleus A+1) respectively, $G=81.4/[g\cdot A^{1/3}]$ g is a adjustable parameter in exciton model.

In this work, the measured (n,2n) cross sections have been collected as complete as possible, analyzed and evaluated for about 98 nuclei in the mass range of A=78-209. Fitting these expressions to available data by using a least-squres method, the systematic parameters could be extracted and fixed at k=2.105, m=21.676 taking g=325.

At present, with a set of parameters g, k, m and above formulae (2), (5) and (7-9), the (n,2n) and (n,3n) cross sections in the range from threshold to 24 MeV have been predicted in the given precision.

Results and Discussions

The calculated results are partly shown in Figs.1-6. All of these results were carried out using above systematic formulae. In these Figs.,some measured data are also shown[6-8]. From Figs.,we can see that.

(1) The cross sections of six nuclei from ^{93}Nb to ^{209}Bi were predicted for E between the (n,2n) threshold (B_1) and (n,4n) threshlod (B_3). The predicted cross sections show excellent agreement with measured data within errors. These results indicate that the systematic formulae (7) have a generality and the parameters g, k, m are reliable.

(2) When A is lighter, the predicted cross sections are always lower than measured data (see Fig. 1 ^{93}Nb). With increasing of A the agreement is better. It also indicates that the formulae (7) is farily good for heavy mass nuclei (see Figs. 4, 5, ^{181}Ta and ^{197}Au). However with the exception of ^{209}Bi (see Fig. 6), it seems to be related with the shell effect of the level density parameter a.

(3) When $E>B_1$, near by the vicinity of the (n,2n) threshold the curve is going up rapidly with increasing of E. Predicted cross sections are higher than measured data. The reasons are mainly due to neglect of the competition from gamma-ray emissions and a deficiency of the level density parameter a. A simple correction is adopted with the so-called "effective thresholds" method. In the region of 14-15 MeV, the rise of the curve with E increase becomes relatively slow and seems to have a plateau, then we can see that the agreement between predicted and measured cross sections is quite satisfactory. When $E>B_2$, in spite of the (n,3n) reaction occurs, but due to the contribution of preequilibrium emission, the (n,2n) curves start to increasing than before, and the (n,3n) curves drop down correspondently at its increase. Thus the better agreement between predicted and measured cross sections has been also obtained than before taking only the evaporation model. Obviously, this is a quite well improvement. However, in any case the reliability of the results for the (n,3n) reaction isn't good enough to compare with the (n,2n) reaction.

As mentioned above, we can get the following conclusions:

The systematic behaviour of (n,2n) and (n,3n) cross sections could be still described by using a set of parameters g, k, m, even if the contribution of pre-equilibrium emission has been taken into accout in the formulae of evaporation model.

With this set of parameters g=325, k=2.105, m=21.676 and above formulae, the (n,2n) and (n,3n) cross sections could be predicted for the unmeasured and hardly measured nuclei in the range of medium and heavy mass ones. that's exactly what nuclear data scientists require today.

If the (n,4n) reaction and the processes of the outgoing charged-particles are considered, the consistence between predicted and measured cross sections could be also improved to a great extent.

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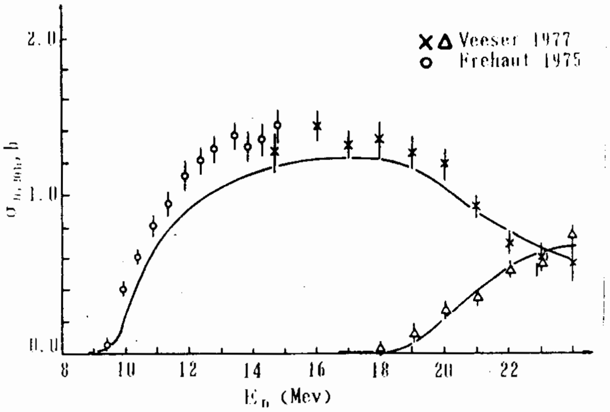

Fig. 1 ^{93}Nb(n, 2n), (n, 3n) Reaction Cross Sections

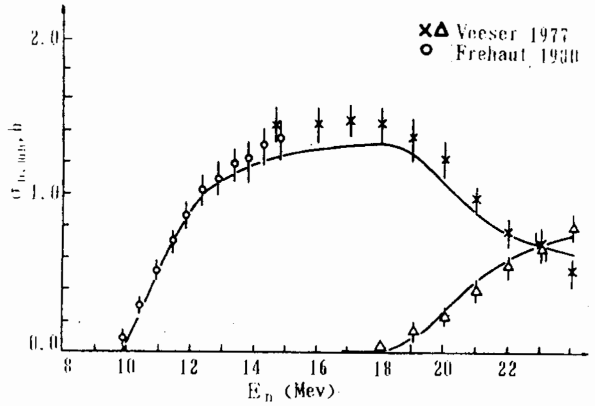

Fig. 2 ^{103}Rh(n, 2n), (n, 3n)Reaction Cross Sections

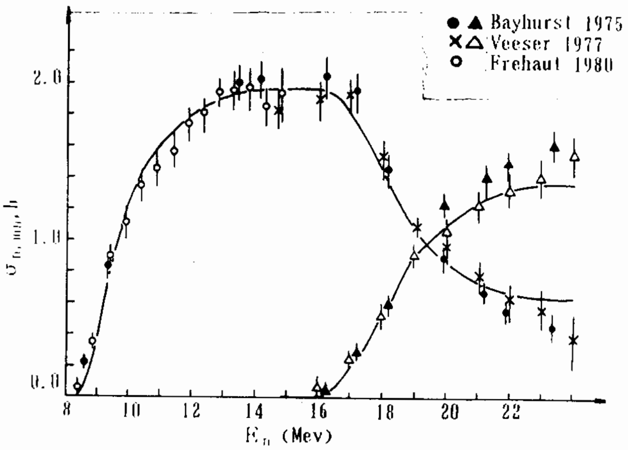

Fig. 3 ^{169}Tm(n, 2n), (n, 3n)Reaction Cross Sections

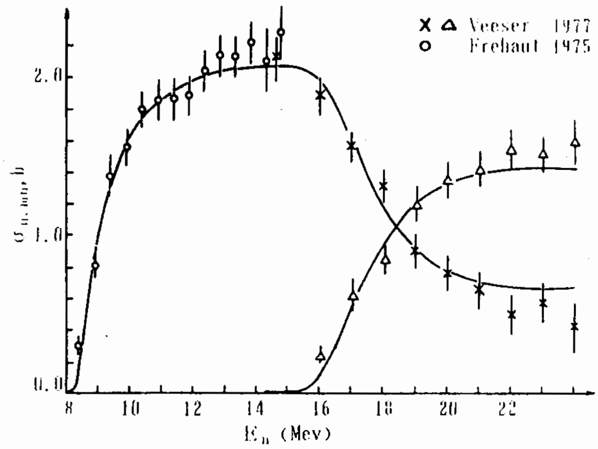

Fig. 4 ^{181}Ta(n, 2n), (n, 3n)Reaction Cross Sections

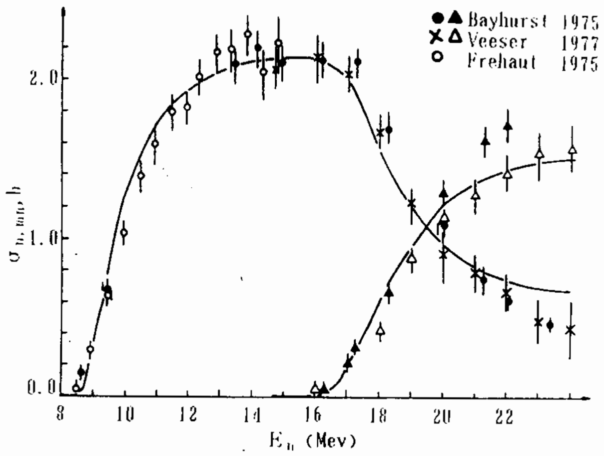

Fig. 5 ^{197}Au(n, 2n), (n, 3n)Reaction Cross Sections

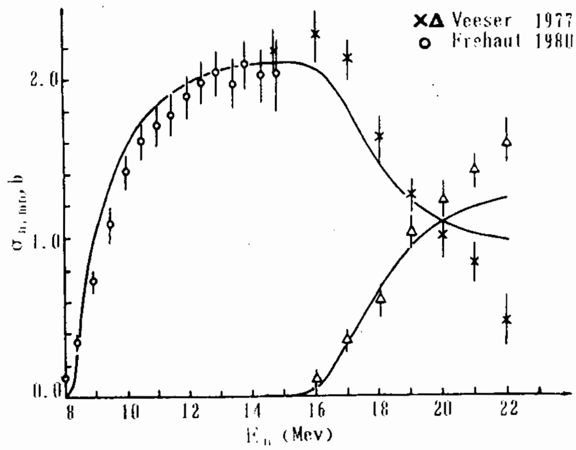

Fig. 6 ^{209}Bi(n, 2n), (n, 3n)Reaction Cross Sections