

A METHOD FOR PREDICTION OF PROMPT FISSION NEUTRON SPECTRA

Anatoli F. Grashin and Michael V. Lepeshkin

Moscow Engineering Physics Institute,
Moscow 115409, USSR

Abstract: Three-parameter formula for the prompt-fission-neutron integral spectrum is derived from a thermodynamical model. Two parameters, scission-neutron weight $p=11\%$ and anisotropy factor for accelerated fragments $b=10\%$, are determined from experimental data, the same values being assumed for any type of fission. The thermodynamical theory provides the value of the third parameter, temperature τ , thus prognosing neutron spectrum and average energy with an error about 1%.

(prompt fission neutrons, scission neutrons, anisotropy factor)

The Thermodynamical Model

Earlier/1/ authors suggested a formula for the prompt-fission-neutron integral spectrum with pre-equilibrium effects for fully accelerated fragments taken into consideration. The formula provides a good fit in the energy region $E \geq 1$ MeV, however theoretical curve is consistently lower than experimental points when $E \rightarrow 0$ (see dashed line in fig.1). This discrepancy can be removed

with scission-neutron-emission being taken into account within the framework of the same thermodynamical model. The scission-neutron effect is reported in refs./2,3/.

Assuming the dependence of neutron emission on the angle θ in the fission-fragment CMS to be

$$1 + b \cos^2 \theta, \tag{1}$$

we obtain for laboratory spectrum

$$N(E) = (1-p_s)N(E; \tau, \alpha, \beta, E_f) + p_s N(E; \tau, \alpha_s, \beta \rightarrow 0, E_f \rightarrow 0), \tag{2}$$

$$N(E; \tau, \alpha, \beta, E_f) = N_0 (e^{-x} - e^{-y}) + \beta N_0 \Phi(E) / 4 E_f (1 + \beta/3),$$

$$\Phi(E) = \frac{\tau}{2\alpha} \left[(x^2 + 2x + 2 - \alpha^2) e^{-x} - (y^2 + 2y + 2 - \alpha^2) e^{-y} \right] - 2(E - E_f/3)(e^{-x} - e^{-y}) + \frac{(E - E_f)^2}{\tau} \left\{ e^{-\alpha} [E_1(x - \alpha) - E_1(y - \alpha)] - e^{\alpha} [E_1(x + \alpha) - E_1(y + \alpha)] \right\}. \tag{3}$$

Here $E_1(x) = \int_x^\infty e^{-\xi} d\xi / \xi$,

$$x^2 = \alpha^2 + 2\alpha(\sqrt{E} - \sqrt{E_f})^2 / \tau,$$

$$y^2 = \alpha^2 + 2\alpha(\sqrt{E} + \sqrt{E_f})^2 / \tau,$$

$$N_0^{-1} = 2(2\alpha\tau E_f)^{1/2} K_1(\alpha).$$

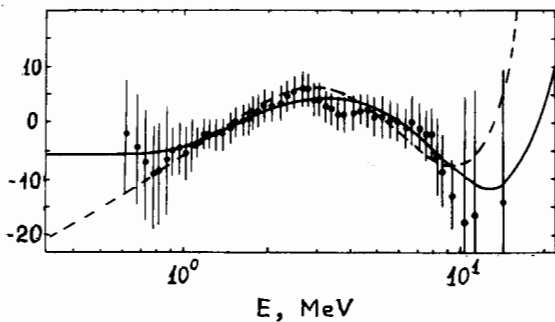


Fig.1 Percentage deviations of the U-235(0.53 MeV) spectra from Maxwellian with $T_M = 1.321$ MeV. Solid line corresponds to set 5 from Table 1, dashed line is obtained without scission-neutron emission/1/.

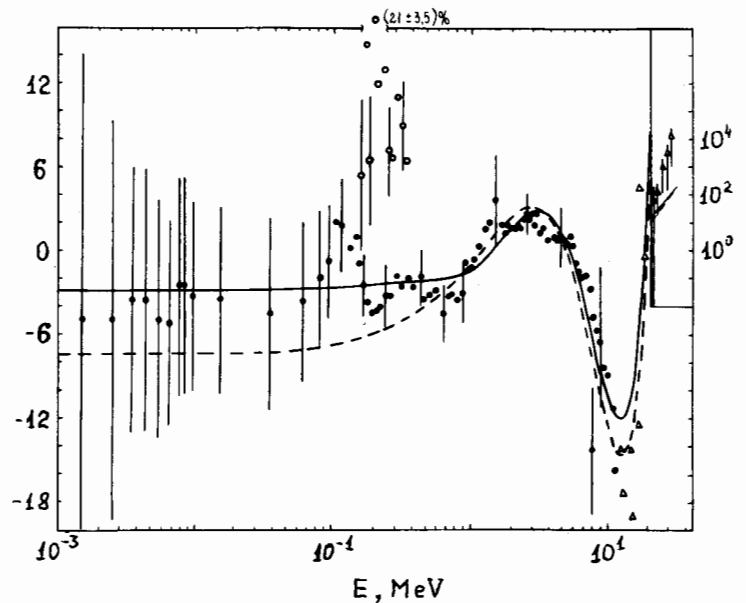


Fig.2 The same as in fig.1, but for Cf-252 (sf), $T_M = 1.42$ MeV. Solid line corresponds to set 1 in Table 1, dashed - to set 2. Data: ● ref./4/, △ ref./6/, ○ ref./10/.

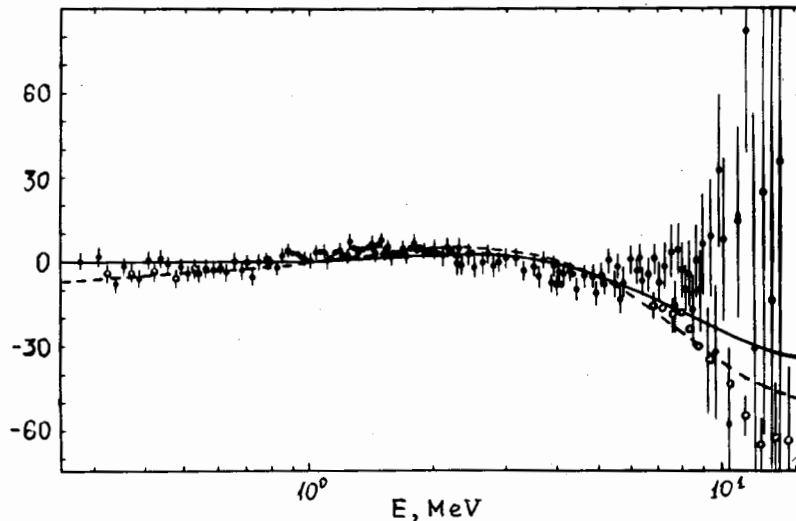


Fig.3 The same as in fig.1, but for Pu-239(n,f) with $T_M=1.438$ MeV. Solid curve corresponds to set 7 in Table 1, dashed - to set 8. Data: ● ref./9/, ○ ref./8/ for $E \leq 0.525$ MeV and $E \geq 6.792$ MeV.

A parameter p_s in expression (2) is the scission-neutron weight, E_f - is the fission-fragment kinetic energy per nucleon, pre-equilibrium parameters are functions of fragment mass number A and compound-nucleus mass number A_F :

$$\alpha = \alpha_0 A^{-1/2}, \quad \alpha_s = \alpha_0 A_F^{-1/2}. \quad (4)$$

The first term in eq.(3) is the expression derived earlier/1/ for the spectrum. The CMS neutron anisotropy is described by the second term in eq.(3). Spectrum (2) is normalized to unity and corresponds to the mean neutron energy

$$\langle E \rangle = (1-p_s) [1.5\tau K_2(\alpha)/K_1(\alpha) + E_f] + 1.5p_s \tau K_2(\alpha_s)/K_1(\alpha_s). \quad (5)$$

Analysis of Experimental Data

The formulas derived were used to analyse data for Cf-252(sf)/4,6/, U-235+n(0.53 MeV)/7/, Pu-239+n(0.53 and 0.215 MeV)/8,9/ with $\alpha_0=303$ as the best fit

to the experiment. The values of fit parameters τ , p_s , b are listed in Table 1, deviations of spectra(2) from Maxwellian distributions $N_M(E, T_M)$ with appropriate temperatures T_M are presented in figs.1-3. The spectrum of Cf-252(sf) which is obtained from a large body of experimental data over the wide energy interval 0.0003-28 MeV, gives the reliable indication of scission-neutron contribution and CMS neutron anisotropy. It is noteworthy that from the integral laboratory spectrum result the same parameters p_s and b , which were determined from difficult multi-parameter measurements in the CMS of fragments/3/. For the mean scission-neutron energy we have

$$\langle E \rangle = 1.5\tau K_2(\alpha_s)/K_1(\alpha_s) = 1.46(5) \text{ MeV}$$

that coincides with the value 1.5(3) MeV in ref./3/.

We performed also fitting data from ref./5/ which are a part of information utilized in ref./4/. This additional

Table 1. Parameter sets for spectrum (2), with asterisk are the parameters not varied in fitting

N ^o	experiment	τ MeV	p_s %	b %	E_f MeV	$\langle E \rangle$ MeV	χ^2/DF
1.	Cf-252(sf) /4,6/	0.904(3)	11.2(1.1)	10(3)	0.784	2.132(5)	0.38
2.	Cf-252(sf) /5,6/	0.901(6)	10.3(1.2)	2(3)		2.134(6)	0.43
3.		0.905(3)	11.2*	10*		2.134(5)	0.55
4.	U-235+n(0.53 MeV) /7/	0.823(5)	10.3(1.8)	10*	0.800	2.021(8)	0.24
5.		0.824(3)	11.2*	10*		2.016(5)	0.24
6.	Pu-239+n(0.215 MeV) /9/	0.884(5)	12.1(1.1)	10*	0.803	2.108(6)	1.19
7.		0.879(3)	11.2*	10*		2.107(5)	1.20
8.	Pu-239+n(0.53 MeV) /8/	0.836(8)	1.0(1.8)	10(6)	0.801	2.115(8)	0.37
9.		0.877(3)	11.2*	10*		2.102(5)	1.40

Table 2. Calculated parameters and mean energies (in MeV) for fission induced by thermal and reactor neutrons. For Pu-239(th) averaged values from analysis of data /8/ and /9/ are given.

nuclide	τ (th)	E_f	$\langle E \rangle$	τ (τ)	E_f	$\langle E \rangle$
Th-229	0.751	0.775	1.88	0.736	0.771	1.93
-232				0.803	0.774	1.96
U -233	0.795	0.800	1.97	0.830	0.796	2.02
-235	0.803	0.802	1.985	0.833	0.794	2.025
-236				0.837	0.794	2.03
-238				0.845	0.794	2.045
Np-237	0.82	0.803	2.01	0.845	0.795	2.045
Pu-239	0.875(3)	0.804	2.101(5)	0.899	0.795	2.13
-240				0.901	0.784	2.125
-241	0.875	0.797	2.095	0.900	0.789	2.13
-242				0.894	0.792	2.12
Am-241	0.906	0.805	2.15	0.93	0.796	2.18
Cm-245	0.935	0.791	2.185	0.96	0.785	2.22
Cf-249	0.973	0.786	2.24	1.00	0.782	2.28

analysis provides the same values of p_g and $\langle E \rangle$ within errors - see sets 2, 3 in Table 1. Thus p_g , $\langle E \rangle$, and τ are not sensitive to experimental data change. However, the varying of parameter b is not worth when we have not got reliable information in the region $0 < E < 0.4$ MeV.

Data for U-235+n(0.53 MeV) and Pu-239+n(0.215 MeV) are well fitted with formula (2) and values $p_g=11.2\%$, $b=10\%$

which are obtained for Cf-252(sf). Data from ref./10/ have unreliable bump in the energy region $E < 0.35$ MeV(see fig. 2), and fitting with this bump would give a lowered mean energy $\langle E \rangle = 2.120$ MeV.

A big dip at $E=12$ MeV in the spectrum from ref./8/ also may not be considered reliable, the dip having essential influence on the analysis of data. Fitting the data/8/ with fixed parameters (set 9 in Table 1) leads to a high value $\chi^2/DF=1.4$ because of points in the energy range $E > 10$ MeV, whereas $\chi^2/DF=0.7$ for $E < 10$ MeV. The fit is practically the same as for Knitter's data/9/, the curve passing through the dip region between the points from ref./8/ and ref./9/ - see solid line in fig.3. Having extrapolated parameters N^o9 from Table 1 to the thermal fission, we obtain the mean energy $\langle E \rangle = 2.095(5)$ MeV which coincides with the value 2.087(15) MeV from ref./4/.

Prediction of Spectra

The results obtained indicate that two parameters p_g and b may be considered the same for any case of fission. A third parameter τ is related to a temperature T from the thermodynamical fission model/11/ in the following manner: $T=T_0 + \tau$. Substituting values of T obtained from fission-product mass distributions and using the approximation

$$T_0 = 1.009 + 0.004(350 - A_F - Z_F) \text{ MeV, (6)}$$

we can make spectrum prediction for

arbitrary compound-nucleus (A_F, Z_F). The values of calculated parameters with errors $\Delta\tau \approx \Delta E_f \approx 0.01$ MeV, $\Delta\langle E \rangle \approx 0.02$ MeV for some thermal- and reactor-neutron-induced reactions are listed in Table 2. For all the cases $p_g=0.112$ and $b=0.1$, and we can neglect a change in parameters(4) substituting $\alpha=27.8$ and $\alpha_g=19.6$. Formula (5) can be written in the simple form

$$\langle E \rangle = 1.585\tau + 0.888 E_f \quad (7)$$

with an accuracy about 0.001 MeV.

Formulas (2),(3) are applicable in excitation energy region $E^* < 6$ MeV where emission channel (n,nf) is closed. Transformation to different excitation energies may be performed with derivatives $\Delta\tau/\Delta E^* = 0.018 - 0.012$; $\Delta\langle E \rangle/\Delta E^* = 0.027 - 0.014$ as in previous publications /1/.

REFERENCES

1. A.F. Grashin: *Atomnaya Energiya* **58**, 59(1985); *Radiation Effects* **93**, 37 (1986)
2. P. Riehs: *Acta Phys. Austr.* **53**, 271 (1981)
3. E.A. Seregina: *Yadernaya Fizika* **42**, 1337(1985)
4. B.I. Starostov: VANT, *Yadernye Konstanty*, vypusk 3, 16(1985)
5. M.P. Poenitz: *Proc. Int. Conf. Nucl. Data Sci. Techn.*, Antwerp, 465(1982)
6. H. Marten: *ibid.*, p.483
7. P.I. Johansson: *Nucl. Sci. Eng.* **62**, 695(1977)
8. *idem*; AERE-R-8636(Harwell, 1977), App. A, tab. by J.M. Adams
9. H.-H. Knitter: *ibid.*, tab. by J.M. Adams
10. J.M. Boldeman: *Nucl. Sci. Eng.* **93**, 181(1986)
11. A.F. Grashin: *Izvestiya AN SSSR, ser. fiz.* **49**, 188(1985)