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THE VARIABLE MOMENT OF INERTIA MODEL IN TERMS OF NUCLEAR SOFTNESS

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Abstarct: The variable moment-of-inertia (VMI) model for the ground-state bands in even-even nuclei is expressed in terms of the various orders of "nuclear softness", that were introduced for the first time in the nuclear softness (NS) model. The range of validity of this, so-called, VMI(NS) model is found to remain the same as that of VMI model. Also the softness parameter defined in VMI model is shown to be the same as the first order softness in VMI(NS) model. Retaining the softness to first and second orders only, denoted, respectively, as VMI(NS2) and VMI(NS3), the model is applied to all the even-even nuclei whose ground-state band is observed experimentally upto 6^+ state or more. Significantly improved fits to the experimental data are obtained.

(Angular-momentum dependent moment-of-inertia, VMI model, various orders of "nuclear softness", NS model)

Introduction

It is now well established experimentally /1/ that Yrast (grond -state) bands can be expected throughout the Periodic Table for all the even -even nuclei away from closed shells. Theoretically, several models /2-5/ have been introduced for correlating such a data, without distinguishing for the much accepted rotational or vibrational nature of the nuclei. In this attempt, the variable moment-of-inertia (VMI) model of Mariscotti et al /2/ is one of the earliest and very successful. This model uses two parameters (the ground-state moment-of-inertia $\mathcal{I}_0$ and the restoring force constant C), obtained by the numerical solution of two coupled equations. The excitation energy of the state J in this model is defined as sum of the rigid rotational energy with moment-of-inertia $\mathcal{I}$ varying with angular momentum J and a potential energy term, harmonic in angular momentum dependent moment-of-inertia $\mathcal{I}_J$ about its mean (ground-state) value $\mathcal{I}_0$.

A variant of the VMI approach is the nuclear-softness (NS) model of one of us /3/, where the nucleus is treated to belong to total "nuclear-softness" regime. Here the nucleus is considered simply as a rigid rotator with its moment-of-inertia $\mathcal{I}$ varying with angular momentum J. Thus, by allowing the various orders of softness i.e. relative increase of $\mathcal{I}$ with J, this model was also applied successfully to all the even-even nuclei, including even the nuclei having only two particles or holes with respect to a closed shell. The predictions of the two-parameter NS model (called NS (2), with $\mathcal{I}_0$ and first order softness σ_1 as its parameters) are very much identical but that of three -parameter NS model (the NS(3), with an additional order of softness, σ_2) are better than those of VMI model.

More recently, based on the predictions of the interacting boson model (IBM1), Klein and his associates /5/ have advanced two generalizations of the VMI model, namely: the variable anharmonic vibrator model (VAVM) and the generalized VMI (GVMI) model. Both models use three parameters each and modify the rotational energy term (the J(J+1) dependence) in VMI model expression. The VMI model is shown to be a special case the GVMI model and the predictions of VAVM are in better agreement with experiments as compared to those of GVMI or VMI model. This work has created a renewed interet /6/ in the VMI model hypothesis.

In this paper, we extend the NS model /3/ concept to the VMI model /2/. In other words, we translate the VMI model into the language of the NS model and introduce the various orders of "nuclear-softness" in the VMI model. It may be mentioned that whereas the two parameters of VMI model are related /2/ to the first order softness ($\sigma_1 = \hbar^2/2C\mathcal{I}_0^3$), the NS model calculations /3/ show that the first order nuclear softness is required even for the strongly deformed nuclei. Hence the inclusion of second (and higher)-order nuclear softness is a must in such analysis. The NS model thus supports the view /7/ that one /two additional parameters are required for an extended success of VMI formalism.

The Model

The enengy expression of the VMI model /2/ is

$$E = \frac{\hbar^2}{2\mathcal{I}_J}J(J+1)+\frac{1}{2}C(\mathcal{I}_J-\mathcal{I}_0)^2 \tag{1}$$

Following the NS model /3/, we make the Taylor series expansion of $\mathcal{I}_J$ in eq. (1) about its ground state value $\mathcal{I}_0$ for J=0. This would give us the VMI expression in terms of the various orders of "nuclear softness", defined /3/ as

$$\sigma_1 = \frac{1}{\mathcal{I}_0}\frac{\partial \mathcal{I}_0}{\partial J}$$

$$\sigma_2 = \frac{1}{2!\,\mathcal{I}_0}\frac{\partial^2 \mathcal{I}_0}{\partial J^2}$$

$$\sigma_3 = \frac{1}{3!\,\mathcal{I}_0}\frac{\partial^3 \mathcal{I}_0}{\partial J^3}\text{, etc.} \tag{2}$$

These corrections arise due to the variation of $\mathcal{I}$ with J. Collecting and rearranging the various terms in σ_i (i=1,2,3,....), the Taylor series expression of eq. (1) gives

$$E_J = \frac{\hbar^2}{2\mathcal{I}_0}J(J+1)\left(\frac{1}{1+\sigma_1 J+\sigma_2 J^2+\sigma_3 J^3+\dots}\right) + \frac{1}{2}C\,\mathcal{I}_0^2\,J^2\left(\sigma_1+\sigma_2 J+\sigma_3 J^2+\dots\right)^2 \tag{3}$$

For the restoring force constant C=0, eq. (3) reduces to the NS model expression of Ref. 3, as expected. Therefore, we refer to this model as VMI(NS).

The Range of Validity of VMI(NS) Model

The range of validity of this model (eq. (3)) remains the same as that of the original VMI expression (1). Remembering that $\sigma_2, \sigma_3, \ldots$ are the second, third and higher order corrections, this can be demostrated by keeping the nuclear softness to first order alone in eq. (3),

$$E_J = \frac{\hbar^2}{2\mathcal{J}_0}\frac{J(J+1)}{1+\sigma_1 J} + \frac{1}{2} C \mathcal{J}_0^2 J^2 \sigma_1^2 \tag{4}$$

Then, imposing the equilibrium condition

$$\frac{\partial E_J}{\partial \sigma_1} = 0 \tag{5}$$

on eq. (4), we get

$$\frac{\hbar^2}{2\mathcal{J}_0}(J+1) = C \mathcal{J}_0^2 \sigma_1 (1+\sigma_1 J)^2 \tag{6}$$

Since the range of nuclear softness is given by

$$0 \leq \sigma_1 \leq \infty \quad , \tag{7}$$

the combined result of the equations (4) and (6) in the two limits of σ_1 , give

$$E_J(\sigma_1 = 0) = \frac{\hbar^2}{2\mathcal{J}_0} J(J+1) \tag{8}$$

$$E_J(\sigma_1 \to \infty) = \frac{3}{2} C^{1/3} \mathcal{J}_0^{2/3} \left[\frac{\hbar^2}{2\mathcal{J}_0} J(J+1)\right]^{2/3} \tag{9}$$

In tems of the energy ratios $R_J = E_J/E_2$, the range the validity of the VMI(NS) model is thus obtained, from equations (8) and (9), as

$$\left[\frac{1}{6} J(J+1)\right]^{2/3} \leq R_J \leq \frac{1}{6} J(J+1) \tag{10}$$

which is exactly the same as for the VMI model. For the case of J=4, eq. (10) gives

$$2.23 \leq R_4 \leq 3.33 \tag{11}$$

Also, for the particular case of J=0, eq. (6) reduces to

$$\sigma_1 = \frac{\hbar^2}{2C\mathcal{J}_0^3} \tag{12}$$

which is exactly the definition of "softness" deduced in VMI model.

Table 1. Experimental and Calculated Energy levels (in MeV) of ground-state bands in even-even nuclei. Only $J \geq 8^+$ are given since $J=2^+, 4^+$ and 6^+ are used in fitting the parameters of the models.

	J^π	8^+	10^+	12^+	14^+	16^+	18^+
^{120}Xe	Expt	2.0489	2.8700				
	VMI(NS2)	2.1223	2.9707	3.9415			
	VMI(NS3)	2.0489	2.6927	3.2887			
	VMI	2.009	2.713	3.469			
	NS(3)	2.1549	3.1272	4.4125			
	VAVM	2.101	2.889	3.749			
^{122}Xe	Expt	2.2173	3.0389	3.8195			
	VMI(NS2)	2.2427	3.1552	4.2036	5.3877		
	VMI(NS3)	2.2173	3.0528	3.9489	4.8839		
	VMI	2.110	2.857	3.659	4.512		
	NS(3)	2.2797	3.3344	4.7482	6.7351		
	VAVM	2.220	3.066	3.993	4.988		
^{126}Ba	Expt	2.0889	2.9423	3.7475	4.4197	5.2451	(6.1947)
	VMI(NS2)	2.1133	3.0507	4.1440	5.3925	6.7961	8.3544
	VMI(NS3)	2.0889	2.9505	3.8906	4.8841	5.9082	6.9425
	VMI	1.968	2.720				
	NS(3)	2.1415	3.1944	4.5940	6.5243	9.3387	
	VAVM	2.087	2.948				
^{128}Ce	Expt	1.8201	2.5312	3.1077	3.6683	4.3582	
	VMI(NS2)	1.8462	2.6677	3.6194	4.6999	5.9083	7.2441
	VMI(NS3)	1.8201	2.5591	3.3424	4.1416	4.9317	5.6917
	VMI	1.762	2.469				
	NS(3)	1.8491	2.6890	3.6969	4.9056	6.3643	8.1451
	VAVM	1.823	2.581				
^{152}Sm	Expt	1.12537	1.60934	2.1489	2.7363		
	VMI(NS2)	1.1329	1.6398	2.2250	2.8872	3.6255	
	VMI(NS3)	1.12537	1.6063	2.1359	2.7011	3.2902	
	VMI	1.0960	1.5485	2.0474	2.5871	3.1631	
	NS(3)	1.1288	1.6251	2.1932	2.8340	3.5506	
	VAVM	1.1201	1.5912	2.1105	2.6714	3.2689	

Table 1. continued....

	J^{π}	8^+	10^+	12^+	14^+	16^+	18^+
^{154}Sm	Expt	0.9031	1.3333	1.8262			
	VMI(NS2)	0.9120	1.3688	1.9142	2.5479		
	VMI(NS3)	0.9031	1.3317	1.8194	2.3558		
	VMI	0.897	1.321	1.805	2.343		
	NS(3)	0.9033	1.3313	1.8159	2.3447		
	VAVM	0.901	1.327	1.810	2.346		
^{158}Er	Expt	1.4932	2.076	2.6808	3.190	3.663	4.229
	VMI(NS2)	1.5125	2.1505	2.8828	3.7085	4.6273	5.6387
	VMI(NS3)	1.4932	2.0708	2.6812	3.3053	3.9269	4.5332
	VMI	1.4430	1.9895	2.5828			
	NS(3)	1.5231	2.2047	3.0500	4.1145	5.4878	7.3193
	VAVM	1.494	2.081	2.722			
^{166}Yb	Expt	1.09829	1.6059	2.1757	2.7795	3.2740	3.7831
	VMI(NS2)	1.1092	1.6517	2.2943	3.0362	3.8770	4.8164
	VMI(NS3)	1.09829	1.6055	2.1747	2.7916	3.4429	4.1161
	VMI	1.0940	1.5998	2.1738	2.8083		
	NS(3)	1.0961	1.5954	2.1470	2.7336	3.3401	3.9538
	VAVM	1.0955	1.5978	2.1638	2.7850		
^{172}Hf	Expt	1.0375	1.5213	2.0648	2.6543	3.2775	3.9199
	VMI(NS2)	1.0470	1.5642	2.1785	2.8895	3.6968	4.6003
	VMI(NS3)	1.0375	1.5237	2.0740	2.6758	3.3176	3.9881
	VMI	1.0358	1.5220	2.0766	2.6921	3.3625	4.0829
	NS(3)	1.0343	1.5090	2.0334	2.5901	3.1637	3.7414
	VAVM	1.0343	1.5137	2.0557	2.6521	3.2970	3.9854
^{176}W	Expt	1.140	1.648	2.206	2.801	3.425	4.002
	VMI(NS2)	1.1483	1.6911	2.3246	3.0470	3.8570	4.7539
	VMI(NS3)	1.140	1.6540	2.2254	2.8390	3.4807	4.1374
	VMI	1.144	1.666	2.256	2.905	3.609	
	NS(3)	1.1324	1.6223	2.1456	2.6804	3.2126	3.7301
	VAVM	1.136	1.643	2.209	2.827	3.490	
^{186}Os	Expt	1.4209	2.0681	2.7814	3.4395	3.9340	4.4934
	VMI(NS2)	1.4236	2.0829	2.8340	3.6660	4.5697	5.5370
	VMI(NS3)	1.4209	2.0703	2.7983	3.5873	4.4207	5.2828
	VMI	1.3877	2.0029	2.6927	3.4480		
	NS(3)	1.4250	2.0894	2.8520	3.7050	4.6422	5.6590
	VAVM	1.4165	2.0574	2.7775	3.5664		
^{232}Th	Expt	0.5569	0.8270	1.1374	1.4833	1.8595	2.2634
	VMI(NS2)	0.5577	0.8321	1.1525	1.5159	1.9193	2.3603
	VMI(NS3)	0.5569	0.8282	1.1420	1.4932	1.8772	2.2892
	VMI	0.5576	0.8292	1.1434	1.4963	1.8843	2.3046
	NS(3)	0.5551	0.8196	1.1175	1.4396	1.7773	2.1230
	VAVM	0.555	0.821	1.125	1.463	1.832	2.227
^{244}Cm	Expt	0.5014					
	VMI(NS2)	0.5019	0.7581				
	VMI(NS3)	0.5014	0.7559				
	VMI	0.503	0.759				
	NS(3)	0.5003	0.7506				
	VAVM	0.500	0.750				

Calculations and Discussion of Results

We have applied our model to all the even-even nuclei ($\sim$150 nuclei) whose ground- state bands are known experimentally upto the 6^+ state or more. The softness parameters in eq. (3) are kept to only first and second orders, thereby reducing the number of parameters to three ($\mathcal{J}_0$, σ_1, C) and four ($\mathcal{J}_0$, σ_1, σ_2, C), respectively. The results of these two calculations are denoted as VMI(NS2) and VMI(NS3), respectively, in the following. Based on the viewpoint that bands are build from the "ground- state up" /5/, the parameters are obtained by fitting the states upto 6^+ or 8^+ (3 or 4 parameteres).

In table 1 we present the results of only a few representative nuclei. For comparisons the results of the VMI, NS(3) and VAVM models are also given. The predictions of GVMI and NS(2) are not included in Table 1, since VAVM gives better results than GVMI and NS(2) gives results almost identical to VMI model results. A careful comparison shows that the predictions of our VMI(NS2) are already an improvement over the VMI model. Since, the nuclear softness to first-order is required ($\sigma_1 \neq 0$) even for strongly deformed nuclei, it is more realistic to investigate the prediction of VMI(NS3) for the softer nuclei. Though the parameters have now increase to four, the predictions of VMI(NS3) are in much closer agreement with the experiments and also an improvement over the three- parameters NS(3) model. This means that for very soft nuclei, either one has to allow nuclear softness to orders higher than the second-order or that it is not enough to attribute the effects of the variation of moment-of- inertia with angular momentum to the rigid rotational term alone. This point is being investigated further.

Finally, we have also analyzed the systematics of the parameters. We observe some significant differences with the VMI, VAVM and GVMI model parameter systematics. The details will be published elsewhere.

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