Measurement of Double Differential Cross Sections of Charged Particle Emission Reactions by Incident DT Neutrons

－Correction for Energy Loss of Charged Particle in Sample Materials－

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In the measurement of charged particle emission spectrum induced by neutrons, correcting the energy loss of charged particle in sample materials becomes a very important inverse problem. To deal with this inverse problem, we have applied the Bayesian unfolding method to correct the energy loss, and tested the performance of the method. Although this method is very simple, it was confirmed from the test that the performance was not inferior to other methods at all, and therefore the method could be a powerful tool for charged particle spectrum measurement.

1. Introduction

Correction for the energy loss of charged particle in sample materials is a serious problem in measurement of charged particle emission spectrum induced by neutrons. Recently, a new simple method was developed for unfolding various measured radiation spectra by extending Bayes theorem [1]. In the present study, we have, thus, tried to apply this method to our experimental data of charged particle emission cross-section measured by the E-TOF two-dimensional analysis at OKTAVIAN of Osaka University [2].

In the past, our experimental data (double differential charged particle emission cross section (DDXc) induced by DT neutrons) were corrected by the conventional method based on an assumption that any charged particle passes half thickness of sample material. This method was applicable if only the energy loss could be suppressed to very small value by using a thin sample foil. For example, we used 25 μm and 10 μm thick foil for proton and α-particle measurements, respectively. However, recently elements that cannot be supplied as an enough thin sample are planning to be used for cross section measurement, like light elements. In this study, focusing on a measurement in which there exists a remarkable energy loss in the sample material as a result of using a thick sample foil, we developed an unfolding code based on the Bayes theorem to accurately correct the energy loss.

2. The New Method

The Bayes' unfolding method is applicable to almost all kind of radiation measurement analysis. There are some variations in the Bayes' estimation method. In this work, *spectrum type Bayes estimation method* has been applied to our charged particle spectrum unfolding problem. In the method, estimation calculation can be realized using just measured pulse height spectrum (histogram) data.

The estimation procedure is expressed as

$$est_j^{(l+1)} = \sum_i^m \left(d_i \times \frac{est_j^{(l)} \times r_{ij}}{\sum_j^n est_j^{(l)} \times r_{ij}} \right) \qquad ,(j = 1, n) \qquad (1)$$

where r_{ij} called *likelihood* is the response of the detection system which provides the probability of a detection event giving pulse height h_i for the charged particle energy E_j. d_i (i=1,m) is the detected pulse height spectrum. $est_j^{(l)}$ (j=1,n) is the estimated spectrum revised by lth estimation calculation. The above formula is repeatedly used for the pulse height spectrum d_i in this work. Revised $est_j^{(l)}$ is used for prior information to the next revise calculation.

3. Application and verification

In order to verify the above method in the charged particle spectrum unfolding problem, several analyses were carried out using numerical simulation and practical measured results, i.e., α-particles spectrum measurement and its unfolding for ^{27}Al samples (60mm ϕ × 10 μm, 25 μm, 100 μm thick) with incident DT neutrons.

3-1. Evaluation of Response Function

In order to carry out unfolding, the response function should be prepared. They are evaluated on the assumption that energy loss occurs only in sample material not in the detector. It means that response, in this case, is made considering energy loss in the sample. The response function is calculated by using SRIM (TRIM-96) code. The obtained response function is shown in Fig. 1.

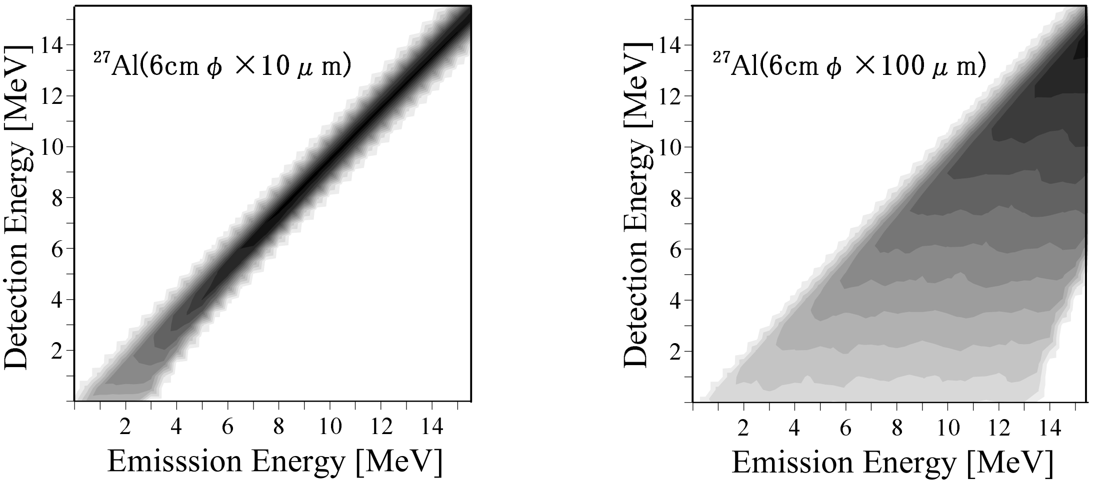

Fig. 1 Response Function for ^{27}Al(n,xα) measurement calculated by SRIM (θ =45deg.).

3-2. Verification Using Numerical Simulations for ^{27}Al Sample

The verification of the new unfolding method was confirmed through numerical simulation using infinite statistic pseudo-measured spectrum (folded spectrum; $\vec{d}$), which was given as

$$\vec{d} = \boldsymbol{R} \cdot \vec{p} \tag{2}$$

where $\boldsymbol{R}$ is the response function. $\vec{p}$ is the ideal spectrum quoted from the evaluated nuclear data(JENDL-FF). Fig. 2 shows the unfolded results. In these figures, an excellent spectrum reproduction is seen even if the diagonality of response function is not good, i.e., in case of 100 μm in thickness. In such a case, more iteration is required to reproduce the ideal spectrum than thinner ones. For evaluating reproducibility, we introduce the amount of information entropy, ***B***, which is defined as

$$B = \sum_{j=1}^{n} p_j \log\left(\frac{est_j}{p_j}\right) \tag{3}$$

where p_j(j=1,n) is the ideal spectrum. est_j(j=1,n) is the unfolded spectrum. Unfolding performance is

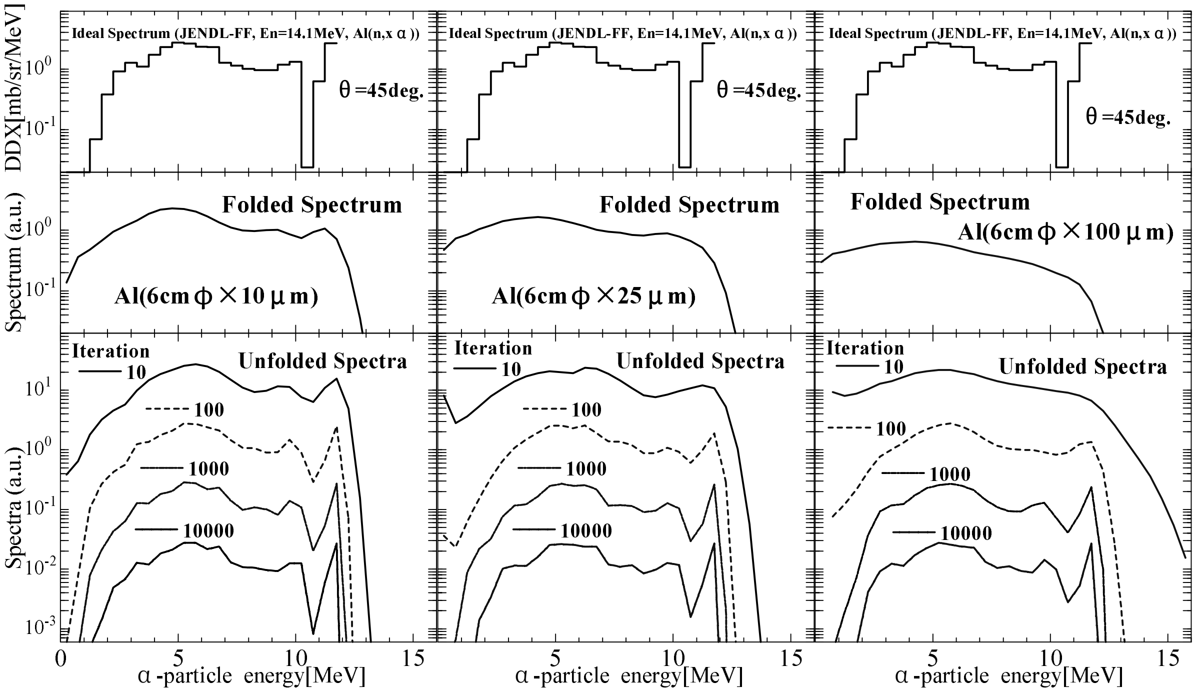

Fig. 2 Unfolding tests using infinite statistic spectra.

estimated by this value. The larger the ***B*** value becomes, the better reproduction of the ideal spectrum is realized. The relation between the ***B*** value and the number of iteration is shown in Fig.3. It is verified from the figure that the unfolded spectrum approaches the ideal spectrum with increase of the number of iteration, and therefore the ideal spectrum can be reproduced by sufficient iteration even if using a thick sample.

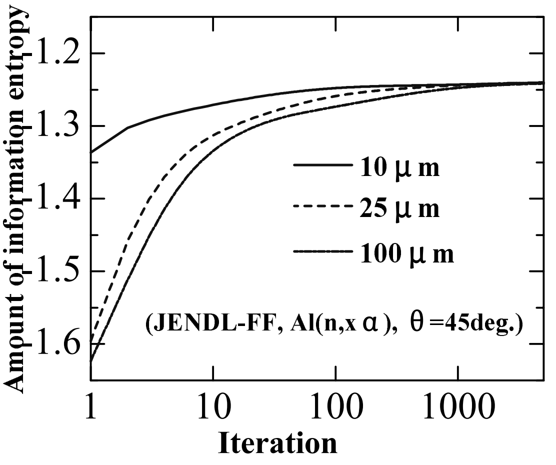

Fig. 3 Behavior of convergence in the case of infinite statistic as a function of the number of iteration.

In the next step, an unfolding test using finite statistic pseudo-measured spectrum, derived from random sampling of infinite statistic folded spectrum (the number of sampling: $\mathbf{N}_{\mathbf{ran}}$), was carried out. The unfolded results are shown in Figs. 4-7. These figures show that this method can reproduce the ideal spectrum to a certain extent. However, as increasing iteration, an unpleasant oscillation is observed as a result of propagation of the statistical fluctuation in the initial folded spectrum, as shown in Figs. 4 and 6. From the Figs. 5 and 7, it is found that there is an optimum iteration number which increases with increase of the statistical accuracy, ($\mathbf{N}_{\mathbf{ran}}$). It can be concluded that an acceptable result can be expected, if $\mathbf{N}_{\mathbf{ran}}$ is set to be an enough large (meaning that a high statistical accuracy is required in actual case.), though in such a case the number of iteration is obliged to increase.

3-3. Verification Using Experimental Results for ^{27}Al Sample

To confirm the conclusion in the previous section, the developed method was applied to a measured spectrum data. Double differential cross section (DDX) of ^{27}Al(n,x α) reaction induced by DT neutrons have been measured at OKTAVIAN of Osaka university. Experiments were carried out

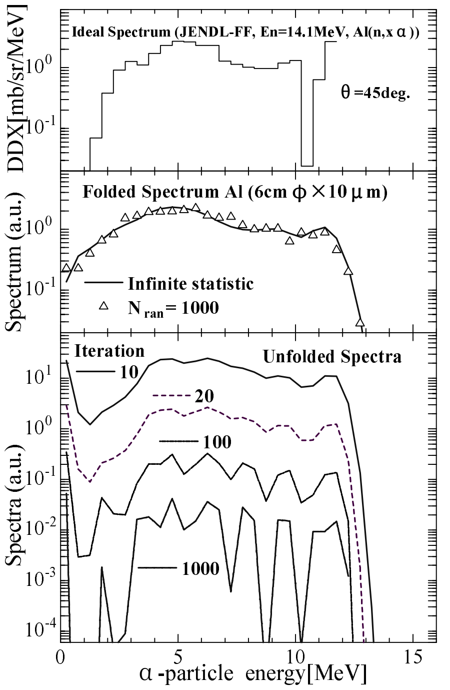

Fig. 4 Unfolding using finite statistic spectra of 1000 counts for ^{27}Al(10 μ m) sample.

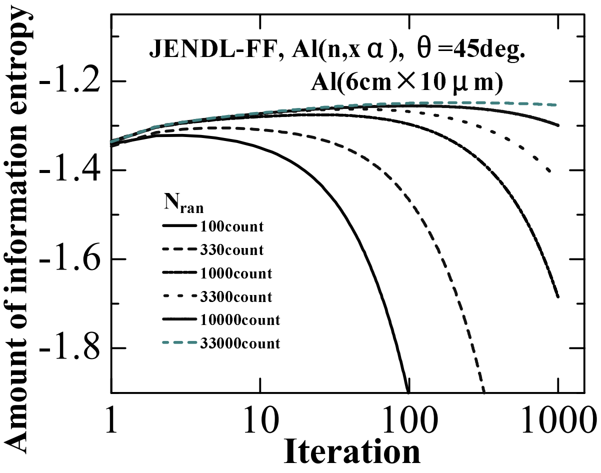

Fig. 5 Unfolding performance using finite statistic spectra of various counts for ^{27}Al(10 μ m) sample.

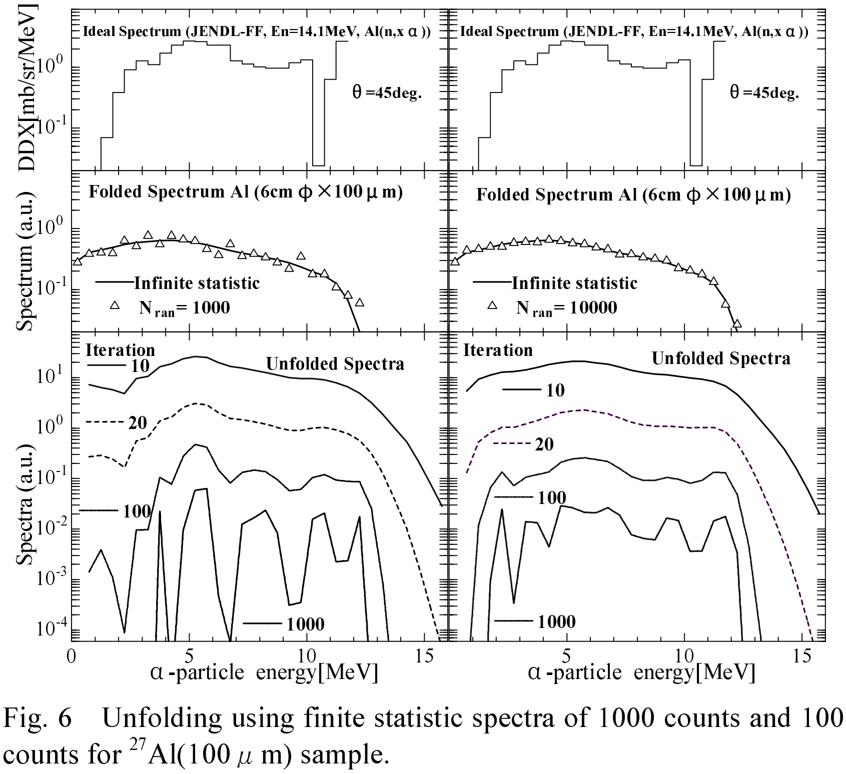

Fig. 6 Unfolding using finite statistic spectra of 1000 counts and 10000 counts for ^{27}Al(100 μ m) sample.

by using the charged particle spectrometer based on the two-dimensional analysis of energy and time-of-flight of emitted charged particle [2]. The schematic arrangement of the spectrometer is shown in Fig. 8. The CsI(Tl) scintillator, 2mm in thickness and 50mm in diameter, was used as a charged particle detector.

Similar to the previous section, three ^{27}Al samples with different thickness (60mm ϕ × 10 μ m, 25 μ m, 100 μ m thick) were used. The measured raw spectra for each sample are shown in Fig. 9. The optimized iteration frequency, that is the revise number of estimation calculation, was evaluated from the amount of information entropy ***B*** calculated using JENDL-FF data. Unfolding was carried out by formula (1) for each spectrum. Fig. 10 shows the respective unfolded spectra (DDX data) together with the corrected spectrum obtained by the conventional method (10 μ m thick sample)[3]. Although the measured raw spectra for three thicknesses show different shape, unfolded spectra present equal data. Further, the unfolded spectrum for 10 μ m shows almost the same as the corrected spectrum with the conventional correction method. It should be noted that the conventional method can not deal with the results for 25 μ m and 100 μ m. These results prove the applicability of this new method to measurements for thicker samples.

4. Discussion

Taking into account the unfolding results using numerical simulation in Sec. 3-2, one can understand the following opposite two requirements exist. One is requirement of thinner sample, because thicker sample makes diagonality of response function worse. The other is thicker sample, because sufficiently accurate result cannot be obtained with a thinner sample. Thus, "*Optimum sample thickness*" exists. The thickness should be determined considering these inconsistent requirements as well as available measurement time.

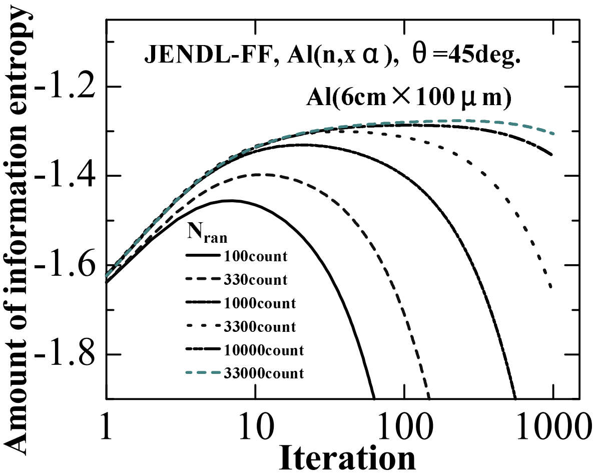

Fig. 7 Unfolding performance using finite statistic spectra of various counts for ^{27}Al(100 μ m) sample.

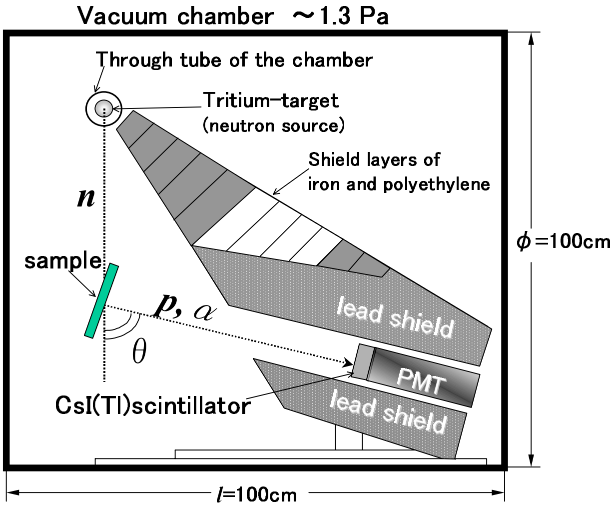

Fig.8 Schematic arrangement of the charged particle spectrometer.

Unfolding tests using the measured DDX spectra show a fairly good agreement among unfolded spectra. However, there is a slight difference with each other. These differences might be caused by selection of the sample thickness, i.e., the number of iteration considering the measurement accuracy, or by the difference between the simulated response function and actual one. Though the difference is mostly within the statistical error, further investigation about it should be performed in the future.

5. Conclusion

The Baysian statistical unfolding method has been applied to correction for energy loss of charged particle in sample materials on measurement of charged particle emission cross section with DT neutrons. And the unfolding applicability has been tested using numerically simulated spectra and practical measured results. From the test results, it is important in the use of the method to determine *optimum condition*, such as sample thickness, iteration frequency and so on. Also the results showed the validity of the method. Although this method is very simple, it was found that the performance was not inferior to other methods at all. Thus, it was confirmed that this method could be a powerful unfolding tool for charged particle spectrum measurement.

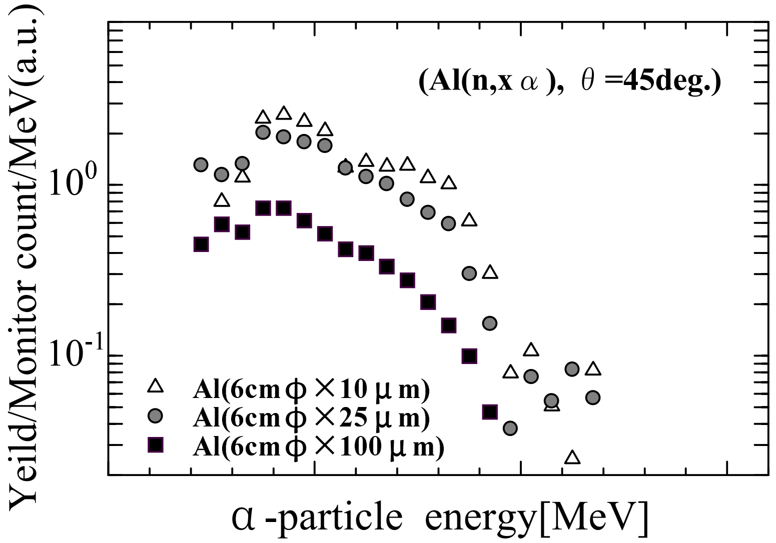

Fig. 9 The measured raw spectra of ^{27}Al(n,x α) reaction for each sample thickness.

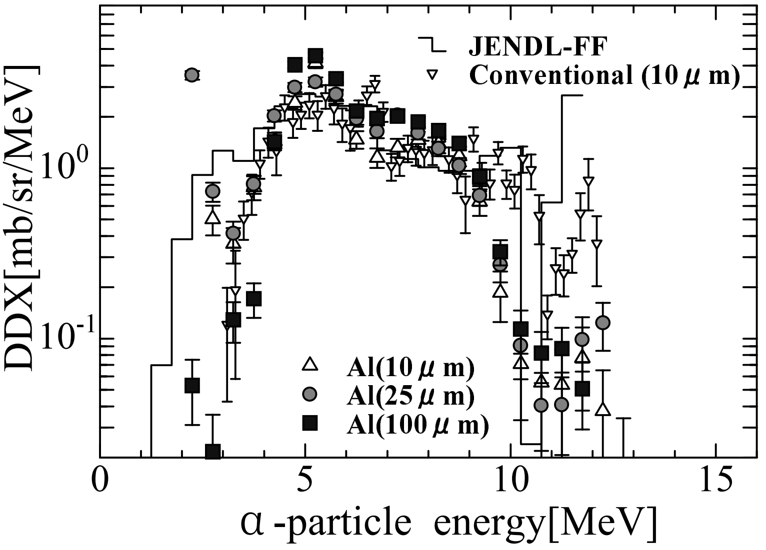

Fig. 10 The unfolded spectra for respective thickness estimated by Bayesian method and corrected spectrum for 10 μ m thick sample obtained by conventional method.

References

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