Calculation of the MSD Two-Step Process with the Sudden Approximation

YOSHIDA Shiro† and KAWANO Toshihiko‡
† *Department of Physics, Tohoku University*
Aoba-ku, Sendai 980-8578, Japan
e-mail: shiro@nucl.phys.tohoku.ac.jp
‡ *Advanced Energy Engineering Science, Kyushu University*
6-1 Kasuga-kouen, Kasuga 816-8580, Japan
e-mail: kawano@aees.kyushu-u.ac.jp

A calculation of the two-step process with the sudden approximation is described. The Green's function which connects the one-step matrix element to the two-step one is represented in r-space to avoid the on-energy-shell approximation. Microscopically calculated two-step cross sections are averaged together with an appropriate level density to give a two-step cross section. The calculated cross sections are compared with the experimental data, however the calculation still contains several simplifications at this moment.

1. Introduction

Quantum mechanical theories of the preequilibrium nuclear reaction have been developed in recent years. There are three well-known statistical multi-step direct (MSD) theories, those are the theories of Feshbach, Kerman, and Koonin (FKK)[1], Tamura, Udagawa, and Lenske (TUL)[2], and Nishioka, Weidenmüller, and Yoshida (NWY)[3]. The FKK theory has a rather simple and feasible formulation in contrast with the TUL and NWY models, and it has been applied to analyses of the medium- and high-energy nuclear reactions. A long-standing problem exists, however, *i.e.* the on-energy-shell approximation in the Green's function made by FKK is inadequate.

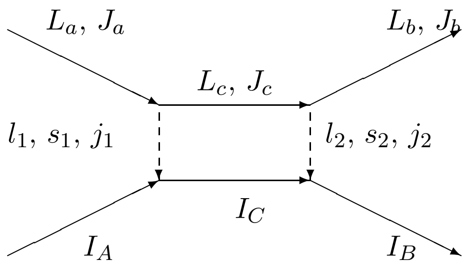
The other two theories employ the different statistical assumptions — the adiabatic and sudden approximations. According to an argument of the time scale of nuclear reactions, an additional particle-hole pair creation is much faster than residual configuration mixing. Therefore the sudden approximation is favored. However no calculation has been done with the NWY model so far, because of its somewhat complicated formulation.

In this study, we calculate the MSD two-step process with the sudden approximation. The nuclear state is expressed by the single-particle shell model. The residual interaction is assumed to be a central Yukawa form with the range of 1 fm. Calculated inelastic scattering cross sections are compared with the experimental data.

2. Microscopic Cross Section of the Two-Step Process

Cross sections of a two-step process in Fig. 1, $A + a \rightarrow C + c \rightarrow B + b$, is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{2step}} = \frac{2I_B+1}{(2I_A+1)(2S_a+1)} \sum_j \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2} \frac{k_b}{k_a} \sum_{m m_b m_a} \left| t_{lsj}^{m m_b m_a} \right|^2 , \tag{1}$$



$$\boldsymbol{l} = \boldsymbol{j} + \boldsymbol{s}, \qquad \boldsymbol{s} = \boldsymbol{s}_1 + \boldsymbol{s}_2$$

$$\boldsymbol{j} = \boldsymbol{j}_1 + \boldsymbol{j}_2 = \boldsymbol{I}_B - \boldsymbol{I}_A = \boldsymbol{J}_a - \boldsymbol{J}_b$$
$$\boldsymbol{j}_1 = \boldsymbol{I}_C - \boldsymbol{I}_A = \boldsymbol{J}_a - \boldsymbol{J}_c$$
$$\boldsymbol{j}_2 = \boldsymbol{I}_B - \boldsymbol{I}_C = \boldsymbol{J}_c - \boldsymbol{J}_b$$

Fig. 1: 2-step process, coupling of angular momenta.

where I is the spin of nucleus, S, L, J are the intrinsic spin, angular momentum, and total spin of nucleon, k the wave number, and l, s, j the transfered angular momentum and spin. The transition matrix element t is given by[4]

$$\begin{aligned} t_{lsj}^{mm_bm_a} &= \sum_{L_aJ_aL_bJ_b} \hat{J}_b\hat{l}\hat{s}\langle L_aS_a0m_a|J_am_a\rangle\langle L_bS_bm,-m_b|J_bm-m_b\rangle \\ &\times \langle J_ajm_b-m, m_a-m_b+m|J_am_a\rangle A^m_{L_aL_b}(\theta)I^{lsj}_{L_aJ_aL_bJ_b}, \end{aligned} \tag{2}$$

where $A^m_{L_aL_b}(\theta)$ is the angle factor, and $I^{lsj}_{L_aJ_aL_bJ_b}$ is the radial overlap integral,

$$I^{lsj}_{L_aJ_aL_bJ_b} = \frac{4\pi}{k_ak_b}\int r_bdr_b\int r_adr_a\chi^{J_b}_{L_b}(k_br_b)h^{lsj}_{L_aJ_aL_bJ_b}\chi^{J_a}_{L_a}(k_ar_a), \tag{3}$$

which contains a radial kernel $h^{lsj}_{L_aJ_aL_bJ_b}$ of the two-step process. When one uses the zero-range approximation, the kernel becomes[5]

$$\begin{aligned} h^{lsj}_{L_aJ_aL_bJ_b} &= \sum_{l_1l_2s_1s_2j_1j_2L_cJ_c} i^{L_a-L_b-l_1-l_2}(-)^{j_1+j_2-j}\hat{j}_1\hat{j}_2^2\hat{s}_1\hat{s}_2\hat{l}_1\hat{l}_2\hat{L}_b\hat{J}_b\hat{L}_c\hat{J}_c^2\hat{I}_C \\ &\times \langle L_cl_100|L_a0\rangle\langle L_bl_200|L_c0\rangle W(J_a\,j_1\,J_b\,j_2\,;J_c\,j)W(I_A\,j_1\,I_B\,j_2\,;I_C\,j) \\ &\times \begin{Bmatrix} L_c & S_c & J_c \\ l_1 & s_1 & j_1 \\ L_a & S_a & J_a \end{Bmatrix}\begin{Bmatrix} L_b & S_b & J_b \\ l_2 & s_2 & j_2 \\ L_c & S_c & J_c \end{Bmatrix} F_{L_cJ_c}(r_b,r_a), \end{aligned} \tag{4}$$

$$F_{L_cJ_c}(r_b,r_a) = f_{l_2}(p_2h_2;r_b)G^{(+)}_{L_cJ_c}(r_b,r_a)f_{l_1}(p_1h_1;r_a), \tag{5}$$

where $G^{(+)}_{L_cJ_c}(r',r)$ is the partial-wave expanded Green's function which connects the one-step matrix element to the two-step one. The Green's function in r-space representation[6] can be calculated as

$$G^{(+)}_{L_cJ_c}(r_b,r_a) = -\frac{2\mu}{\hbar^2k_c}\chi_{L_cJ_c}(k_cr_<)\,\mathcal{H}_{L_cJ_c}(k_cr_>), \tag{6}$$

where $\chi_{L_cJ_c}(k_cr)$ is the distorted wave for the intermediate state, $\mathcal{H}_{L_cJ_c}(k_cr)$ is the out-going wave which is an irregular solution of the Schrödinger equation. These functions have asymptotic forms

$$\chi(k_cr) \sim \{F(k_cr) + C\,[G(k_cr) + iF(k_cr)]\}\,\mathrm{e}^{i\delta_C}, \tag{7}$$
$$\mathcal{H}(k_cr) \sim \{G(k_cr) + iF(k_cr)\}\,\mathrm{e}^{-i\delta_C}, \tag{8}$$

where $G(k_c r)$ and $F(k_c r)$ are the Coulomb functions, and δ_C the Coulomb phase shift.

In Eq. (5), $f_l(r)$ is the form factor which expresses p-h state excitation. We assumed that the particle-hole residual interaction has the Yukawa form with the range of 1 fm. According to an expression of the two-step process with the sudden approximation, an intermediate state is always $1p$-$1h$ state. Therefore the formfactor for the first collision f_{l_1} in Eq. (5) is $\langle 1p1h|V|0\rangle$, and the second formfactor becomes $\langle 2p2h|V|1p1h\rangle$. However we replaced the second one by $\langle 1p1h|V|0\rangle$ for the sake of simplicity. This simplification will be removed in future.

3. Transition to Continuum

Double differential cross sections of the two-step process to the continuum state are calculated as

$$\left(\frac{d^2\sigma}{d\Omega dE_b}\right)_{2\mathrm{step}} = \frac{\mu_a\mu_b}{(2\pi\hbar^2)^2}\frac{k_b}{k_a}\sum_l \sum_{p_1h_1p_2h_2}(2l+1)\hat{\rho}^{(l)}_{p_1h_1p_2h_2}(E_x)\sum_{mm_bm_a}|t^{mm_bm_a}_{l0l}|^2, \qquad (9)$$

where the target spin I_A and the spin transfer s are assumed to be zero, $\hat{\rho}$ is the true level density[7]. As seen in Fig. 2 there are four different paths to arrive the same $2p$-$2h$ state. Amplitudes corresponding to those paths are coherently summed up to give the final two-step amplitude. An example of this rearrangement is shown in Fig. 3, which is the angular distribution of ^{208}Pb(p,p') reaction, for $E_{in} = 22$ MeV, $l = 3$, and the excited $2p$-$2h$ state is $|1f_{7/2}0h_{9/2}(2s_{1/2})^{-1}(1d_{3/2})^{-1}\rangle$ in the Z shell. The optical potential used is the Walter-Guss' potential and the strength of the residual interaction V_0 is taken to be 30 MeV. The thick solid line stands for the coherent sum of four amplitudes which correspond to the different intermediate state. The dot-dashed line is the incoherent sum of the cross sections shown by the thin lines.

Since the $2p$-$2h$ state density in the residual nucleus is very large, it is difficult to calculate Eq. (9) directly. We approximate Eq. (9) by

$$\left(\frac{d^2\sigma}{d\Omega dE_b}\right)_{2\mathrm{step}} = \sum_l (2l+1)\omega(2,2,E_x)R_4(l)\overline{\left(\frac{d\sigma}{d\Omega}\right)_l}, \qquad (10)$$

where $\overline{(d\sigma/d\Omega)_l}$ is the averaged two-step cross section for the angular momentum transfer of l, $\omega(2,2,E_x)$ is the state density of Betak-Dobes[8], $R_4(l)$ the spin distribution.

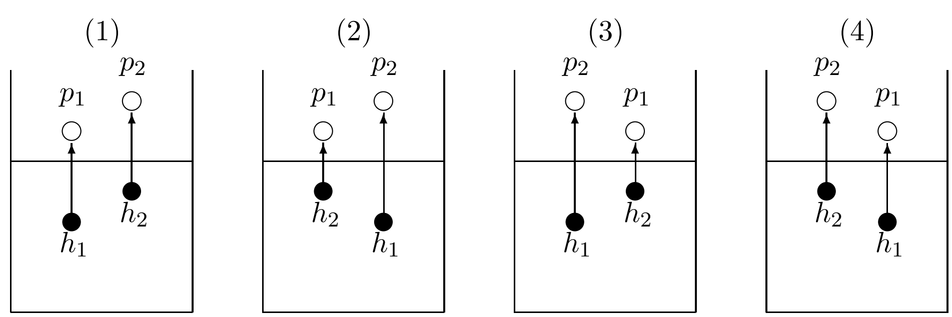


Fig. 2: Four paths to arrive a 2-particle 2-hole state. (1) is the basic configuration, (2) shows an exchange of the two holes, (3) shows an exchange of the two particles, and (4) shows exchanges of the two holes and two particles, respectively.

The average cross section in Eq. (10) is calculated by means of a random sampling of the $2p$-$2h$ state, and the microscopic cross sections for those states are Gaussian averaged. Figure 4 shows various two-step cross sections (light lines) and the average of them (heavy line).

A comparison of the calculated angular distribution of inelastically scattered neutrons from ^{93}Nb for E_{in} = 25.7 MeV and E_{out} =12.5 MeV, with the experimental data[9] is shown in Fig. 5. The dashed line is the one-step cross section, and the dotted line is the two-step one. The effective interaction strength V_0 used was 30 MeV. This result is similar to the NWY calculation of Koning and Akkermans[10], although they made some additional approximations to make calculations easier.

Figure 6 shows a comparison of the angle-integrated energy spectra for the 25.7 MeV neutron incident reaction on ^{93}Nb. The energy spectrum consists of the one-step, two-step, and Hauser- Feshbach components, and the elastic scattering, collective, and $(n, 2n)$ reactions are not included. The sum of one-step and two-step processes well reproduces the experimental data near 15 MeV.

4. Conclusion

We described how the two-step cross sections with the sudden approximation are calculated. At this moment it contains several approximations, such as the replacement of the formfactor at the second collision, use of a phenomenological level density formula, and so on. Such simplifications will be removed in future to give a calculation which is more in line with the NWY theory.

References

[1] H. Feshbach, A. Kerman, and S. Koonin, *Ann. Phys.,* (N.Y.) **125**, 429 (1980).

[2] T. Tamura, T. Udagawa, and H. Lenske, *Phys. Rev. C,* **26**, 379 (1982).

[3] H. Nishioka, H. A. Weidenmüller, and S. Yoshida, *Ann. Phys.,* (N.Y.) **183**, 166 (1988).

[4] G. R. Satchler, *Nucl. Phys.,* **55**, 1 (1964).

[5] N. Hashimoto and M. Kawai, *Prog. Theo. Phys.,* **59**, 1245 (1978); N. Hashimoto, *Prog. Theo. Phys.,* **59**, 1562 (1978).

[6] N. Austern, R. M. Drisko, E. Rost, and G. R. Satchler, *Phys. Rev.,* **128**, 733 (1962).

[7] K. Sato, Y. Takahashi, and S. Yoshida, *Z. Phys. A,* **339**, 129 (1991).

[8] E. Betak and J. Dobes, *Z. Phys. A,* **279**, 319 (1976).

[9] A. Marcinkowski, R. W. Finlay, G. Randers-pehrson, C. E. Brient, R. Kurup, S. Mellema, A. Meigooni, and R.Tailor, *Nucl. Sci. Eng.,* **83**, 13 (1983).

[10] A. J. Koning and J. M. Akkermans, *Phys. Rev. C,* **47**, 724 (1993).

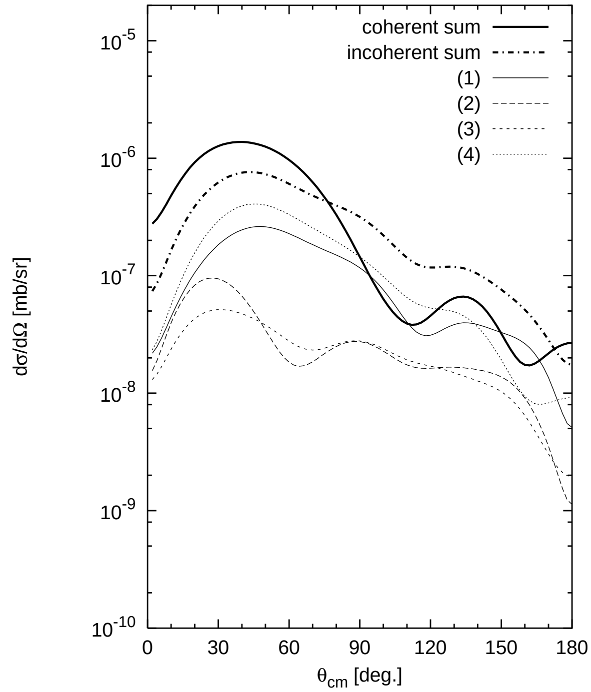


Fig. 3: Microscopic two-step cross sections for $^{208}\mathrm{Pb}(p,p')$, for E_{in} =22 MeV. The $2p$-$2h$ pairs are created in the Z shell. The thin lines are the contributions of each path in Fig. 2, the thick sold line is the coherent sum of the four paths, and the thick dot-dashed line is the incoherent sum.

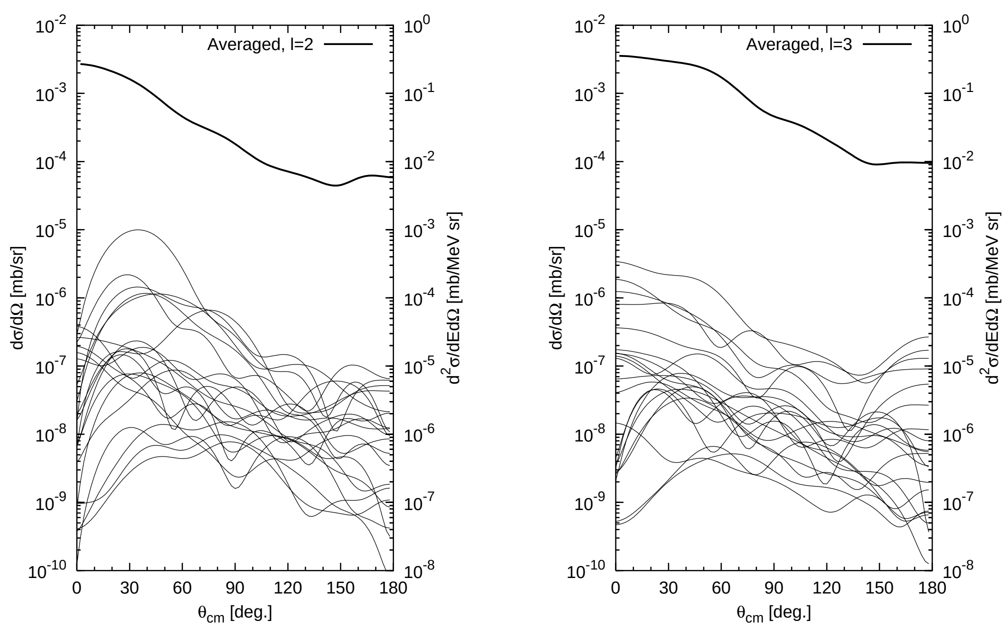


Fig. 4: Averaged microscopic two-step cross sections for the angular momentum transfers of 2 and 3. The heavy solid lines are averaged values multiplied by the state density (on the right axis), and the light lines are some typical microscopic cross sections (on the left axis).

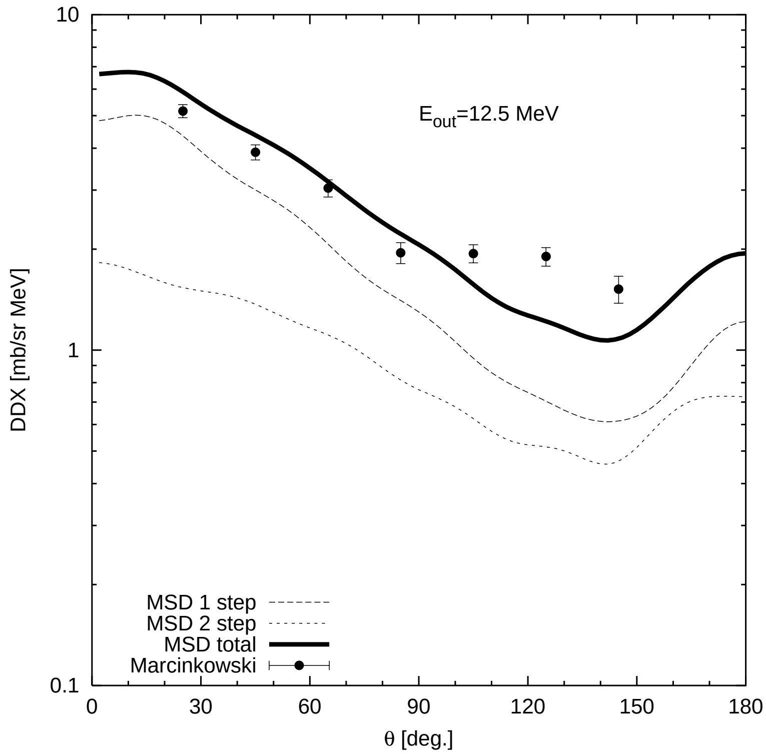


Fig. 5: Comparison of the calculated angular distribution of inelastically scattered neutrons from ^{93}Nb for E_{in} =25.7 MeV, and E_{out} =12.5 MeV, with the experimental data.

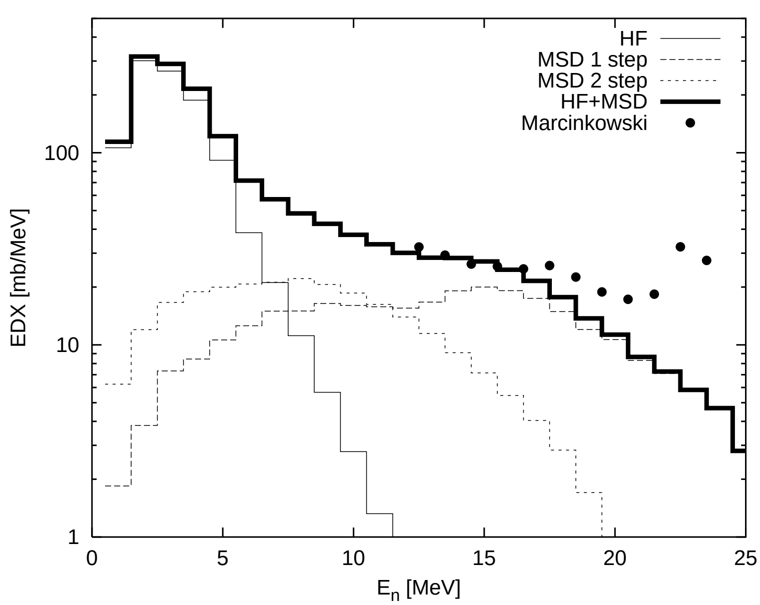


Fig. 6: Energy distribution of the emitted neutrons for the 25.7 MeV neutron incident reaction on ^{93}Nb.