New Formulas for TKE Release in Nuclear Fission Process

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New TKE formulas that will replace the previous existing ones are obtained. Recently, three types of the final deformation of fissioning nuclei were found for actinides out off which only one was observed. The final deformations of the fissioning nuclei was found to be constant and independent of the mass and temperature of the fissioning system. These hence allow to deduce a new formula for the TKE release in nuclear fission process based on the invariance of scission deformations of fissioning nuclei. They yield, TKE(sym) = $0.1173\times(Z_f^2/A_f^{1/3})+7.5$ MeV for the symmetric fission and TKE(asym) = $0.1217\times(Z_f^2/A_f^{1/3})+3.5$ MeV for the asymmetric fission. Details for the new formulas and their comparison with the experimental data are given.

1. Introduction

Mass yield and a kinetic energy of a fission product in fission process of an atomic nucleus are important observable quantities not only for physically understanding fission mechanism but also for practical application in various aspects of nuclear industry. To establish databases for various fission properties is of significant interest and active efforts have been making by the researchers in the nuclear data field. Fission of light elements producing a single symmetric mass yield curve is known, as a nuclide becomes heavy and close to such an element as Ra, the fission yield curve of fragments becomes complicated at low energy and gives a triple humped shape, then a further increase of the mass of a fissioning nucleus leads to an appearance of a double humped mass yield curve. Thus, it was considered that fission of nuclides over the periodic table should show gradually and systematically predictable changes of such properties as mass-yield distribution, total kinetic energy (TKE), etc. But later, as heavy elements were studied, their fission properties always introduced some unexpected results. One of them is the bimodal fission which is characterized by two distinct fission modes for the same symmetric mass division [1]. Also in the spontaneous fission of Fm isotopes, fission properties show dramatic changes [2,3]. They produce very narrow mass yield curves with a very large TKE release which deviate from the prediction of the existing TKE-systematics [4,5] reproducing well the fission observations in a wide region of the relatively lighter nuclides.

In this paper, we report new formulas for the TKE release in nuclear fission processes, which are derived from the scission deformations [6] of fissioning nuclei. Better reproduction of the experimental data by the new TKE formulas than the previously existing ones is indicated.

2. New Formulas for the TKE Release in Nuclear Fission Process

2. 1 TKE formula for the symmetric fission process

For the mass split leading to the average mass of the symmetric fission: $A_1=A_2=A_f/2$, the TKE depends primarily on their mutual Coulomb repulsion of the nascent fragments at scission:

$$TKE_{sym}=e^2\times\rho^2\times(A_f/2)^2/D_{sym}. \qquad (1)$$

Where, ρ is the charge density of the fission fragment and given by the UCD model, i.e. $\rho=Z_f/A_f$, Z_f and A_f being the charge and mass of the fissioning nucleus. The ambiguity brought into the final results by the use of the UCD assumption is less than 1% which has been confirmed via a comparison of the results obtained via UCD charges and via experimental measured charges, respectively[6]. The D_{sym} is the distance between two charge centers of the paired fragments at scission and given by $D_{sym}=\beta_{sym}\times D_0$, β_{sym} is the shape elongation of a nucleus at scission configuration and D_0 is radius of a spherical nucleus. Then the TKE_{sym} is written as,

$$TKE_{sym}=e^2\times(Z_f/A_f)^2\times(A_f/2)^2/[\beta_{sym}\times2\times r_0\times(A_f/2)^{1/3}] \qquad (2)$$

It is further expressed in terms of the conventionally used parameter, $Z_f^2/A_f^{1/3}$,

$$TKE_{sym}=e^2\times[8\times r_0\times(1/2)^{1/3}\times\beta_{sym}]^{-1}\times(Z_f^2/A_f^{1/3}) \qquad (3)$$

Where the term of $e^2\times[8\times r_0\times(1/2)^{1/3}\times\beta_{sym}]^{-1}$ gives a constant of 0.1173 because the β_{sym} is 1.65 for nuclei undergoing the ordinary symmetric fission dominated by the liquid drop like property of the nucleus, and the β_{sym} value is identical and independent from the mass of the fissioning nucleus and its temperature[6]. By applying this functional form, $TKE_{sym} = 0.1173\times(Z_f^2/A_f^{1/3})+b$, to the experimental data, the constant b is determined to be 7.5 MeV. Then, the TKE release in the mass-symmetric fission process is hence obtained.

$$TKE_{sym} = 0.1173\times(Z_f^2/A_f^{1/3})+7.5 \text{ MeV} \qquad (4)$$

In Fig. 1, comparison of the present TKE-function with the available experimental data is shown. Symbols include the experimental data of the present work, the low-energy symmetric fission and the high-energy heavy-ion induced symmetric fission. The experimental data of literature are taken from Refs.[2,4,7-22] and references therein. The result of the presently derived TKE-function is shown by a bold line. The systematics of Viola et al. [4] is displayed by a thin line. The dashed line is the theoretical results from the dynamical calculations by Nix, Davies and Sierk [7,23]. In the calculations, the effects of surface-plus-window dissipation on the dynamical evolution of the fissioning nucleus beyond its fission saddle point was taken into account. In the figure, that the present TKE formula well reproduces the experimental data is seen.

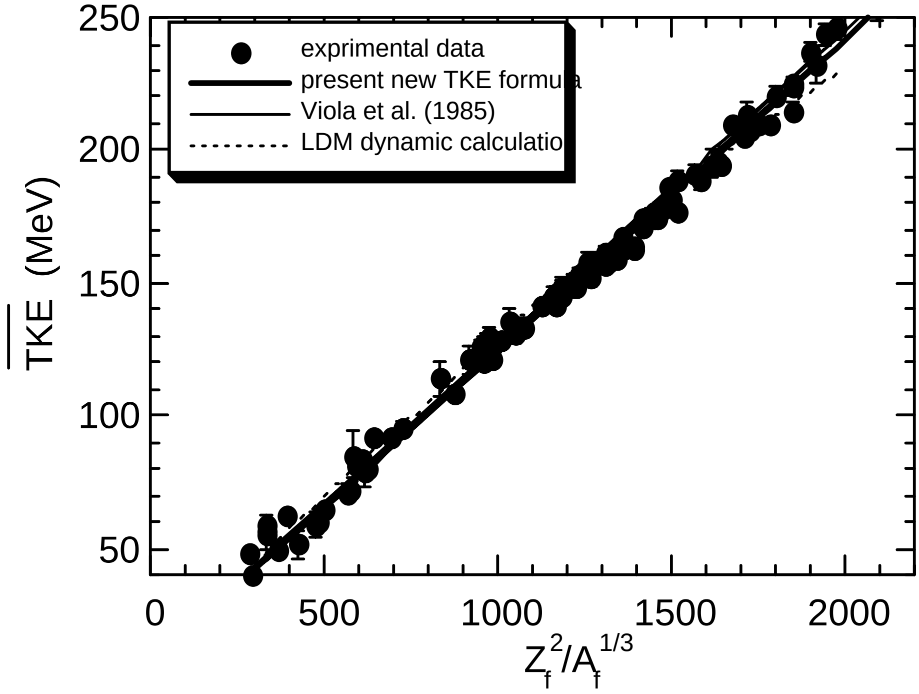

Fig. 1, A comparison of the new TKE formula with experimental data (symbols). The total kinetic energy release in the symmetric fission process is plotted as a function of Coulomb parameter, $Z_f^2/A_f^{1/3}$. Bold line represents the results of the equation (4). The theoretical results by dynamical calculations of liquid drop model are indicated using a dashed line.

2.2 TKE formula for the asymmetric fission process

The mass split leading to the average mass of the asymmetric fission is considered. As being known for several decades that the mean mass for the heavier products of the asymmetric fission is mostly at A_H=140 [24-26]. Hence, the average total kinetic energy can be expressed by,

$$TKE_{asym} = e^2 \times \rho^2 \times 140 \times (A_f - 140)/D_{asym}. \tag{5}$$

From the UCD assumption and $D_{asym} = \beta_{asym} \times D_{0,}$ it is rewritten as a form of,

$$TKE_{asym} = e^2 \times (Z_f/A_f)^2 \times 140 \times (A_f - 140)/[\beta_{a\,sy} \times r_0 \times (140^{1/3} + (A_f - 140)^{1/3})] \tag{6}$$

The term of $(140^{1/3}+(A_f-140)^{1/3})$ for all of nuclei from A_f=227~259 which covers most of the nuclides fissioning asymmetrically can be approximately replaced by $1.587 \times A_f^{1/3}$. The term of $140 \times (A_f-140)/A_f^2$ is approximately equal to a constant of 0.24 for nuclei in the above asymmetric fission region if the uncertainty of ~4% is allowed. Then one obtains,

$$TKE_{asym} = 0.1816 \times \beta^{-1}_{asym} \times (Z_f^2/A_f^{1/3}) \tag{7}$$

The asymmetric shape elongation, β_{asym} was obtained and reported in Ref.[6]. It is a constant value of 1.53 and independent from the fissioning mass, A_f. This gives the coefficient of 0.1217 for equation (7). Thus one obtains the function for the TKE release in the asymmetric fission,

$$TKE_{asym} = 0.1217 \times (Z_f^2/A_f^{1/3}) + 3.5 \text{ MeV} \tag{8}$$

The constant term of 3.5 MeV was obtained from the fitting to the experimental data by applying equation (7) to experimental data measured accurately by the double velocity time-of-flight spectrometers [27-30]. In Fig. 2, the experimental data of the average TKE measured from asymmetric fission reactions are plotted as a function of the parameter $Z_f^2/A_f^{1/3}$. The data indicated by solid circles are from the present measurements in which the observed TKE were decomposed into the independent ones for the symmetric and asymmetric fission [27-30], respectively. The experimental data shown by open squares are taken from Ref. [3] and modified to study very the asymmetric process. The mass-yield curves corresponding to the open squares are all very asymmetric with the peak-to-valley ratios of several tens or even several hundreds. The dashed line is the linear fit by the TKE systematics of Unik et al. [5], the thin line shows the TKE systematic formula of Viola et al. The present TKE function indicated by a bold line very well reproduces the experimental data.

The TKE formula of Viola et al. was first proposed in 1963 [31], then modified in 1966 [20] and further revised in 1985 [4] as the experimental data in a wider range of fissioning nuclei became available. The newest Viola's expression is $0.1187\times(Z_f^2/A_f^{1/3})+7.3$ MeV [4]. From a comparison with the new TKE formulas of equations (4) and (8), it is found that the Viola's TKE function gives values in between the present two functions but closer to equation (4) which is derived from the shape elongation of the symmetric fission. This is a not too surprising result because Viola's formula was obtained from a least-square fitting to the experimental TKE data from fission of nuclides over the chart of the nuclides, most of them fission symmetrically.

3. Conclusion

New formulas for the TKE release in nuclear fission processes were derived. Three types of the final deformation of fissioning nuclei were found for actinides outoff which only one exists. The results indicated that the final deformations of the fissioning nuclei showed to be constant and independent of the mass and temperature of the fissioning system. The identical value of final deformations of fissioning nuclei hence allows one to obtain new empirical TKE formulas: TKE(sym) = $0.1173\times(Z_f^2/ A_f^{1/3})+7.5$ MeV for the symmetric fission and TKE(asym) = $0.1217\times(Z_f^2/ A_f^{1/3})+3.5$ MeV for the asymmetric fission. The present TKE functions show very well reproduction of the experimental data and also provide answers to question why the previous existing TKE systematics couldn’t reproduce the experimental data observed from the asymmetric fission of heavy elements.

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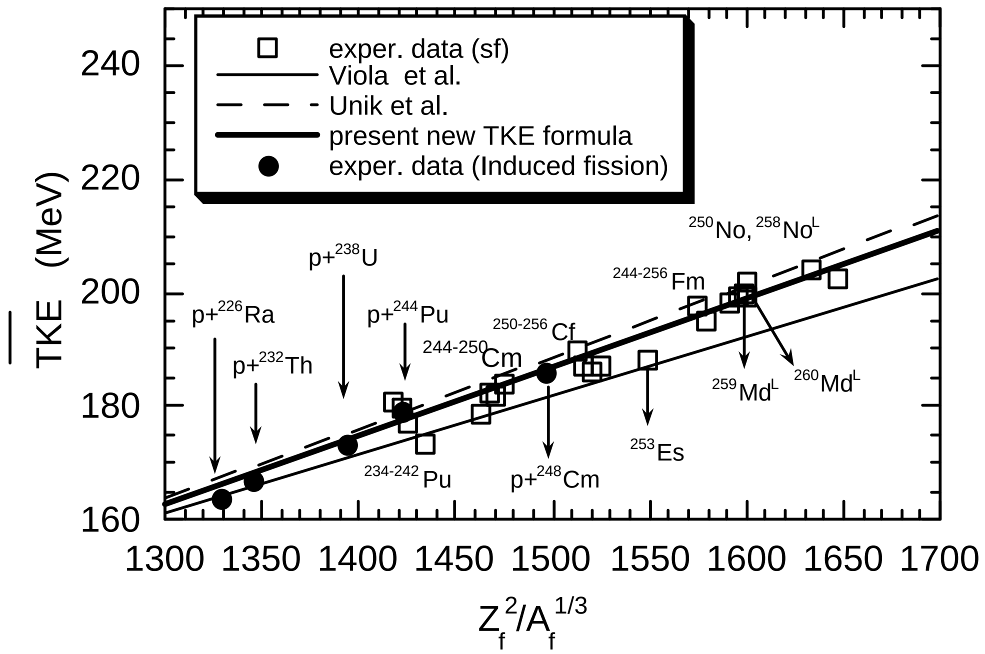

Fig. 2, A comparison of the new TKE formula with experimental data (Symbols). The total kinetic energy release in the asymmetric fission process is plotted as a function of Coulomb parameter, $Z_f^2/A_f^{1/3}$. Bold line shows the results of equation (8). The thin and dashed lines are the results of the previous TKE systematics proposed by Viola and Unik et al., respectively. The fissioning nuclei or systems corresponding to data points are indicated by the names close to data points or far away to the symbols but with arrows.

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