Reactor Kinetics Calculated in the Summation Method and Key Delayed-neutron Data

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The point-reactor kinetics after a step reactivity insertion to a critical condition is solved directly form fission-product (FP) data (fission yields and decay data) for the first time. Numerical calculations are performed with the FP data in ENDF/B-VI. The inhour equation obtained directly from the FP data shows a different behavior at long periods from the one obtained from Tuttle's six-group parameter sets. The behavior is quite similar to the one obtained from the six-group parameter sets in ENDF/B-VI, that were obtained from FP data in a preliminary version of ENDF/B-VI. To identify the erroneous FP data, we examine the asymptotic form of the inhour equation at an infinitely long period. It is found that the most important precursors for long reactor periods are found ^{137}I, ^{88}Br and ^{87}Br. They cover more than 60 % of the reactivity. It is remarkable that ^{137}I alone covers 30-50 % depending on the fissioning system. In addition to the three precursors, ^{136}Te is found a candidate precursor for the peculiarity from the time dependence of the delayed neutron activity. It is recommended that the precision of their Pn values should be improved experimentally. For ^{137}I, ^{88}Br, and ^{87}Br, the relative uncertainty, dPn/Pn, should be decreased down to 2 % and for ^{136}Te to 5%.

1. Introduction

It was pointed out that the reactor kinetics obtained from the ENDF/B-VI delayed-neutron (DN) data shows a peculiar behavior [1]. For analyses of the reactor kinetics, not only the number of the total delayed neutrons but also its time dependence is needed. In principle, these quantities can be calculated from decay and buildup of fission-product (FP) nuclei. From this viewpoint, the DN data in ENDF/B-VI were obtained from the summation calculation with FP data in a preliminary version of ENDF/B-VI.

This paper demonstrates the reactor kinetics calculation directly from the FP decay and buildup for the first time. For simplicity, the calculation is performed for a step insertion after a critical condition. The FP data are taken from ENDF/B-VI so as to identify the source of the peculiarity of the DN data in ENDF/B-VI.

The following notations are use in this paper.

$\bar{\nu}$: average number of total neutrons emitted after a pulse fission

$\bar{\nu}_d$: average number of delayed neutrons emitted after a pulse fission

$\beta = \overline{\nu}_d / \nu$

$n_d(t)$: delayed neutron activity after a pulse fission

k : neutron multiplication = (neutron production)/ (neutron loss)

$\rho = (k - 1) / k$: reactivity

$\Lambda = 1 / k$: mean neutron generation time

2. Kinetics of a point reactor

The kinetics of a point reactor is obtained from an integro-differential equation for the reactor power $p(t)$ in the inverse method with a simple modification.

$$\frac{dp}{dt} = \frac{\rho(t) - \beta}{\Lambda} p(t) + \int_{-\infty}^{t} du \frac{p(u)}{\Lambda} \frac{n_d(t-u)}{\nu} \qquad (2.1)$$

The essential point is to replace the DN kernel by the DN activity after a pulse fission, $n_d(t)$. The DN activity is calculated from the fission yields and decay data in the summation method. Equation (2.1) can be solved formally using Laplace transform.

From Eq. (2.1), we can obtain the inhour equation that relates the reactivity ρ with the asymptotic reactor period T.

$$\rho = \frac{\int_0^{\infty} dt \exp\left(-t / T\right) m_d(t)}{\nu T} \quad . \qquad (2.2)$$

Here, $m_d(t)$ is the DN activity after the infinite irradiation at a constant fission rate 1 fission/s. It is given by

$$m_d(t) = \int_{-\infty}^{0} du \exp n_d(t-u) \quad . \qquad (2.3)$$

It is interesting that the inhour equation (2.2) is written simply with $m_d(t)$. This reflects the fact that the system was critical at $t < 0$.

It is useful to consider an asymptotic formula of Eq. (2.2) for a long $T \left(T \to \infty\right)$. It is given by

$$\rho = \frac{N}{\nu} \frac{1}{T} \qquad (2.4)$$

with $$N = \int_0^{\infty} dt\, m_d(t) \qquad (2.5)$$

being the number of total delayed neutrons emitted after the infinite irradiation.

3. Delayed neutron data in ENDF/B-VI and their summation calculation

In ENDF/B-VI, the time dependence of DN emission is given in terms of parameter values for the following conventional fitting formula

$$n_d(t) = \overline{\nu}_d \sum_{i=j}^{6} \alpha_j \lambda_j \exp\left(-\lambda_j t\right) \quad . \tag{3.1}$$

with $$\sum_{j=1}^{6} \alpha_j = 1 \tag{3.2}$$

Here, α_j and λ_j are the fractional DN yield and the decay constant of group *j*, respectively.

The DN data, $\overline{\nu}_d$, α_j and λ_j , are conventionally evaluated from direct measurements of $\overline{\nu}_d$ and $n_d(t)$. However, ENDF/B-VI obtained these values from summation calculations of $\overline{\nu}_d$ and $n_d(t)$.

The summation calculation simulates decay and buildup of FP's using the FP fission-yield and decay data.

$$\overline{\nu}_d = \sum_{i}^{all\ FP} Y_i P_{ni} \quad . \tag{3.3}$$

Here, Y_i and Pn_i are the cumulative fission yield and the DN emission probability of FP *i*. The time dependence is given by

$$n_d(t) = \sum_{i}^{all\ FP} Y_i P_{ni} \lambda_i \exp\left(-\lambda_i t\right) + \left(decay\ chain\ effect\right) \quad , \tag{3.4}$$

where λ_i is the decay constant of FP *i*. The decay chain effect in Eq. (3.4) reflects the fact that DN precursors can be created in part from decays of their parent FP's. This term is not very large but gives an appreciable contribution to $n_d(t)$.

The summation calculation requires fission yield and decay data of many short lived FP's. However their data were not known precisely enough. This is the reason why the DN data have been evaluated from the direct measurements except for ENDF/B-VI.

Including the decay chain effect, $n_d(t)$ are calculated with an FP decay code in the summation method [2] in the following. The FP fission yield and decay data are taken from ENDF/B-VI.

4. Key precursors for kinetics of a point reactor

The inhour equation (2.2) is calculated with $n_d(t)$ obtained in the summation method. In Fig. 1, it is compared with the conventional calculation obtained from the six-group parameter set evaluated by Tuttle. This figure clearly shows a peculiar behavior at long periods, which is similar to the one obtained from the ENDF/B-VI six-group parameter set [1]. Furthermore, it is seen that the value at the infinitely large period is a good measure of the difference.

DN precursors which contribute much to the reactivity at the infinitely long period can be examined from Eqs. (2.4) and (2.5). They are the ones that give large contributions to *N*, which is the number of the delayed neutrons emitted after the infinite irradiation. Figure 2 shows the contribution from each DN precursor. ^{137}I, ^{88}Br and ^{87}Br cover more than 60 % of the reactivity. It is interesting that the most important precursor is ^{137}I although it has been believed that the longest lived ^{87}Br is the most important at a large period.

To identify DN precursors which cause the peculiar behavior in Fig. 1, the time dependence of $m_d(t)$ is analyzed in the following way because the reactivity ρ is proportional to the Laplace transform of $m_d(t)$ in Eq.(2.2). Specifically, the summation calculation of $m_d(t)$ is compared with the one obtained from Tuttle's six-group parameter set. For simplicity, the difference between the summation calculation and Tuttle's,

$$\Delta m_d(t) = m_d^{\mathrm{Tuttle}}(t) - m_d^{\mathrm{SUM}}(t) \quad , \qquad (4.1)$$

is assumed to be caused by an erroneous FP data of a DN precursor with half life λ. Then, we have $\Delta m_d(t) = c \exp(-\lambda t)$. The graph of $t \times \Delta m_d(t)$ has a peak at $t=1/\lambda$ as a function of t. The lifetime 1/λ corresponds with half life (ln2)/λ. Hence, from the peak time, we can identify a DN precursor whose FP data is questionable.

This prescription is applied to fast fissioning systems of ^{235}U, ^{238}U because the ENDF/B-VI six-group parameters were obtained from the summation calculations for the fast systems. For the fast fission of ^{239}Pu, the summation calculation agrees well with Tuttle's. For ^{235}U(f) in Fig. 3, $t \times \Delta m_d(t)$ has a peak at 36.7 s corresponding to half life 25.4 s. Hence, this difference can be attributed to ^{137}I whose half life is 24.5 s. For ^{238}U, the obtained half life is 20.5 s. So the ^{88}Br (half life 16.5 s) and ^{136}Te (17.5 s) as well as ^{137}I can also be candidates to explain the difference.

5. Precision of fission yields and decay data

To identify key DN data for the peculiar results of the summation calculations, we examine fission yields and decay data of ^{137}I, ^{88}Br, ^{87}Br and ^{136}Te. Here, we include ^{87}Br because it is one of the most important precursors, too. For their half lives, the precision is quite good. It is about 1% or better. Hence, it is expected that the argument in the previous section works well.

Required precision of $n_d(t)$ is evaluated to be 5 % [3]. For simplicity, let us neglect the decay chain effect in Eq. (3.4) and assume all FP half lives are known well. Then, we may consider the following simple relation for the precision.

$$(dPn/Pn)^2 + (dY/Y)^2 = (5\%)^2. \qquad (5.1)$$

If we notice that the fission yields are more difficult to evaluate precisely, we may put

$$dPn/Pn = 3\%, \qquad dY/Y = 4\%. \qquad (5.2)$$

As for Pn values in ENDF/B-VI, however, the relative precision, dPn/Pn, is still large. It is 6 % even for the most important ^{137}I. Uncertainties of fission yields are much larger in ENDF/B-VI.

It is recommended to improve the precision of Pn values of ^{137}I, ^{88}Br, ^{87}Br, and ^{136}Te firstly because precise Pn measurements can now be performed at some laboratories. For the most important precursors ^{137}I, ^{88}Br and ^{87}Br, the precision should be better than 3%, and as good as 2 % to eliminate uncertainties from the Pn values. For ^{136}Te, the precision should be as good as 5 % because this precursor has a relatively smaller contribution than the others.

Once we obtain the precise Pn values, it is much easier to identify remaining erroneous data. It also helps to improve the precision of fission yield values because Pn values are branching ratios of decay chains. Hence, the precise Pn measurement is the first step to make the summation calculation practical for the reactor kinetics calculation.

6. Conclusion

We calculate the reactor kinetics directly from FP fission yields and decay data for the first time. The obtained inhour equation shows peculiar underestimate of reactivity at long periods. It is similar to the one obtained from the DN data in ENDF/B-VI. ^{137}I, ^{88}Br and ^{87}Br are found to cover 60-80% of the reactivity at the infinitely long period. Among them, ^{137}I is the most important. From an additional analysis on the time dependence of the DN activity, we conclude that ^{137}I, ^{88}Br and ^{136}Te could cause the peculiarity of the inhour equation. In order to improve the summation calculation of the reactor kinetics, it is recommended that the precision of the Pn value should be improved to 2 % for the most important ^{137}I, ^{88}Br and ^{87}Br, and to 5 % for ^{136}Te.

[1] D.G.Spriggs, Nucl. Sci. Eng. 114, pp. 342-351, 1993.
[2] K. Oyamatsu, Proc. 1998 Symp. on Nucl. Data, JAERI-Conf 99-002, pp. 234-239, 1999.
[3] private communication with J. Rowlands.

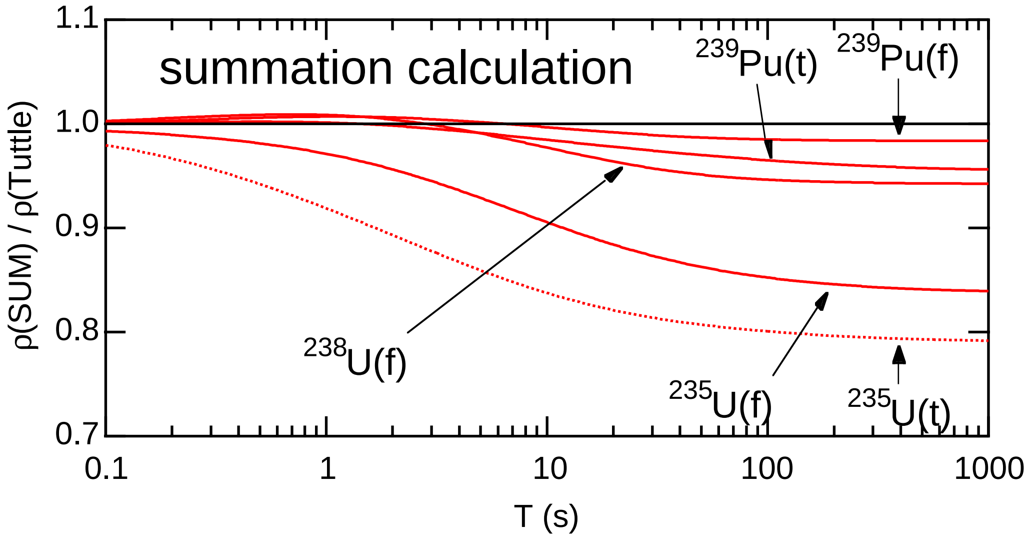

Fig. 1. Comparison of reactivity ρ in dollar between the summation method and the six-group approximation with Tuttle's parameter set.

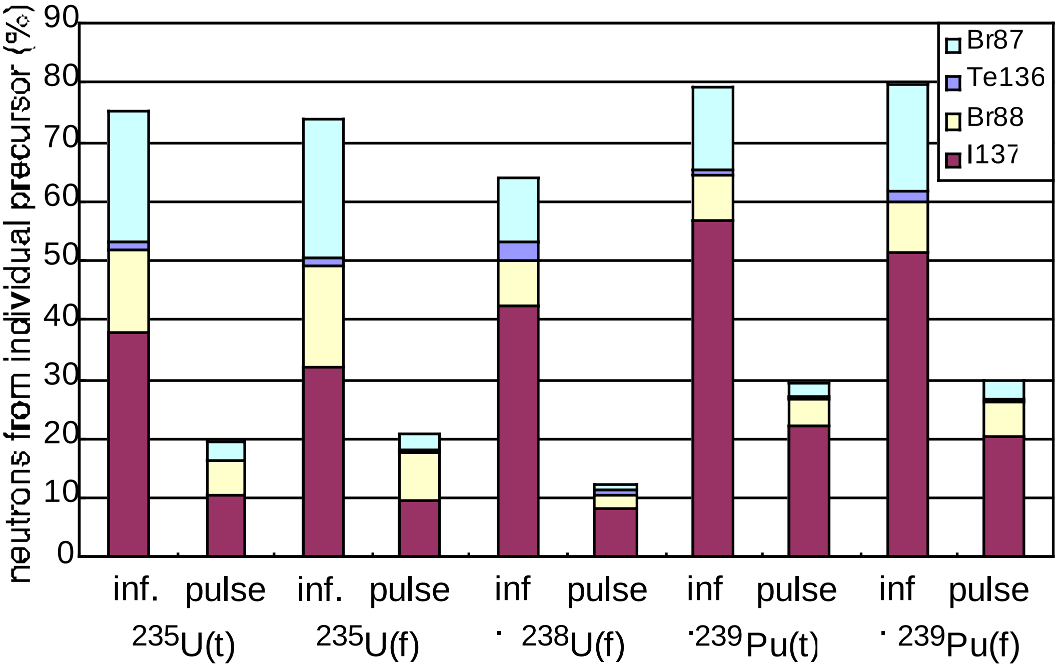

Fig. 2. Delayed neutrons from individual precursors after the infinite irradiation and after a pulse fission. The corresponding precursors in a bar are ^{137}I, ^{88}Br, ^{136}Te and ^{87}Br from the bottom.

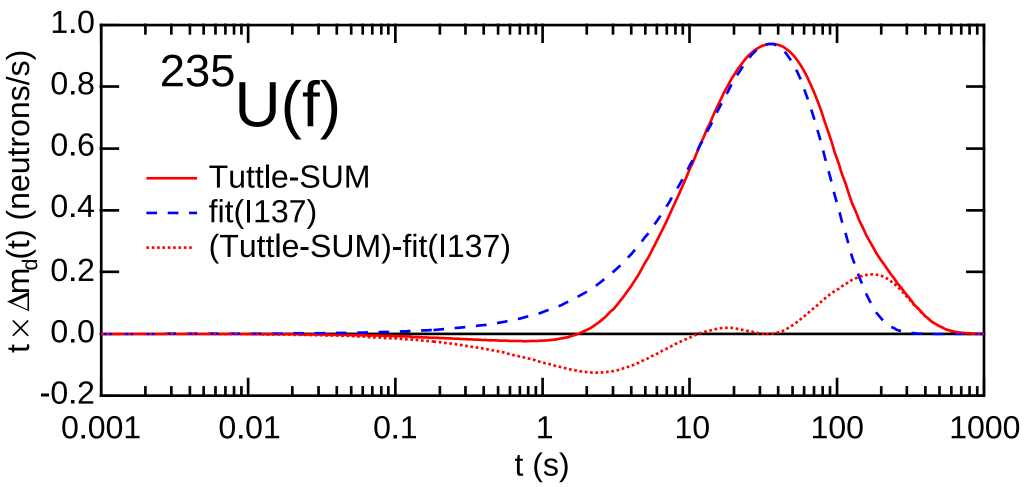

Fig. 3. The difference between summation calculation and Tuttle's. Shown are $t \times \Delta m_d(t)$ and its fitting (adjustment) with ^{137}I.