Unified Model of Nuclear Mass & Level Density Formulas

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The objective of present work is to obtain a unified description of nuclear shell, pairing and deformation effects for both ground state masses and level densities, and to find a new set of parameter systematics for both the mass and the level density formulas on the basis of a model for new single-particle state densities. In this model, an analytical expression is adopted for the anisotropic harmonic oscillator spectra, but the shell-pairing correlation are introduced in a new way.

1. Introduction

In recent years most statistical theory calculations of nuclear reactions have been carried out by using the semiempirical level density formula proposed by Gilbert and Cameron [1] in 1965 , which is based essentially on the Fermi-gas (FG) model and seems to be enough to predict the level densities at a narrow range of excitations. However, it has been in fact well established [2] that the extrapolation of this formula to a wide range of excitation energies is subject to large errors, and that washing out of shell effects should be considered. On the other hand, the energy dependent pairing corrections with the shell-pairing correlation seems to be only correctly considered by means of the microscopic Fermi-gas model or of the Extended Thomas-Fermi plus Strutinsky Integral (ESTFSI) model [3], which are based on the numerical shell-model calculations and also on the BCS formalism. The systematics of nuclear level density depends strongly on the shell,pairing and deformation effects. The nuclear mass formula has been used to determine the "empirical" shell, pairing and deformation energies at the ground state, which are defined as the corrections on the liquid-drop part in the mass formula. The most often used correction energies are those of Myers and Swiatecki(M & S)[4], but the discrepancy in absolute values of shell corrections may amount up to 2 MeV.

The main aim of the present work is to find a new set of parameter systematics for both the mass formula and the level density formula on the basis of the new single-particle state density model. In this model, an analytical expression is adopted for single-particle spectra, but the shell-pairing correlation terms are introduced in a new way [5]. In the next sections typical contents and results of the present model are shown,those are physical meanings of unified model,systematics of parameters for the ground state (mass formula) and for the excited state (level density formula).

2. Basic parameters in unified model

Pysical contents of a unified model are shown as follows：

Pysics of model and parameters

Macroscopic model

Finite range droplet model (FRDM：Berkeley Group, 1995) [6]

Quadrupole deformation : vibration and rotation

Microscopic corrections

Anisotropic harmonic oscillator potential

Single-particle spectrum (analytical expression) :

Shell & Pairing Correlation (*SPC*)

Moments of inertia with pairing-rotation correlation

Systematics of parameters

Asymptotic level density parameter [$a = \alpha A$] A = mass number

Average frequency of oscillator [$\omega = \omega_0 A^{1/3}$]

(α, ω_0) = fitting constants

Single-particle spectrum :

$$g(\varepsilon)=\Sigma_X g_X\{1+\tfrac{1}{3}f_X\Sigma_i\cos\omega_i(\varepsilon-\varepsilon_0)\}\{1-\cos\omega_{PX}(\varepsilon-\lambda_X)\} \quad (1)$$

where $\omega_1=\omega_2=\omega_\perp \approx \omega(1+\frac{1}{3}\delta)$, $\omega_3=\omega_\parallel \approx \omega(1-\frac{2}{3}\delta)$, δ = quadrupole deformation parameter , ω = average frequency = $2\pi/h\omega_{sh}$, ($h\omega_{sh}$= $41A^{-1/3}$), g_X = single-particle level density (2-fold degenerate), f_X = amplitude of shell-structure, ε_0 = main-shell energy, λ_X = Fermi level, ($x \equiv$ p or n), ω_{P_X} = quasi-particle frequency (= $2/\Delta_{0_X}$), $\frac{1}{2}g_X\Delta_{0_X}^2 = 2g_X/\omega_{PX}^2$, Δ_{0_X} = energy-gap at the ground state (from even-odd effects of mass formula). In **Fig.1** shell,pairing and deformation corrections to the droplet terms (FRDM,1995) [7] are shown, which are obtain by means of shell-wise average procedures and of polynomial fitting teckniques. A new mass formula consists from those corrections terms shown in Fig.1. Proton and neutron shells are defined in **Table 1** and there shown are theoretical (fitting) errors of the new formula and of the previous formulas. Among of formulas Present S & P means the case of direct use of simulated data for shell,pairing and deformation corrections from existent experimental mass data.

Temperature dependence of thermodynamic quantities are shown in **Fig.2** to present the assumed behavior of quasi-particle spectrum of a new single-particle spectrum Eq.(2).

3. Systematics of parameters

Values of main parameters, quadrupole deformation parameter δ, α and ω_0 are shown in **Figs.3** and **4**, the later shows effects of collective enhancements for deformed nuclei.

4. Conclusion

The above mentioned results related to systematics of parameters α and ω_0 will give important effects on calculated evaporation spectra. Confirmation of this facts is an objective in future.

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References

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Shell-group		No. of	[σth (MeV)]			
Z-	N-	nuclei	M&S(1967)	M&N(1995)	Present S-P	New formula
20	20	24	6.625	1.477	0.191	0.270
	28	33	9.201	1.363	0.107	0.232
28	28	54	6.528	1.146	0.391	0.415
	50	53	3.969	0.377	0.114	0.250
50	50	192	2.368	0.617	0.329	0.416
	82	267	1.330	0.709	0.318	0.452
82	82	179	1.114	0.479	0.477	0.599
	126	418	0.969	0.484	0.452	0.615
126	126	62	1.118	0.374	0.228	0.719
	184	232	1.302	0.393	0.448	0.534
Total		*1514*	*2.521*	*0.621*	*0.393*	*0.445*

Table 1 Classification into shell-wise groups of nuclei in RIPLE2(1998) experimental mass data and comparison of theoretical fitting error of new mass formulas with those of other previous ones. Present S-P means the results of shell-wise average for shell & pairing corrections by using the FRDM terms by M&N(1995)[6]. Theoretical error σ_{th} is a measure of overall quality representing a precison and is defined by,

$$\sigma_{th}^2 = \frac{1}{\sum w_i} \sum w_i \left[(M^i_{exp} - M^i_{th})^2 - \sigma^{i\,2}_{exp}\right], \qquad (4)$$

$$w_i = \frac{1}{(\sigma^{i\,2}_{exp} + \sigma_{th}^2)^2}$$

and is easily obtained in a few iterations.

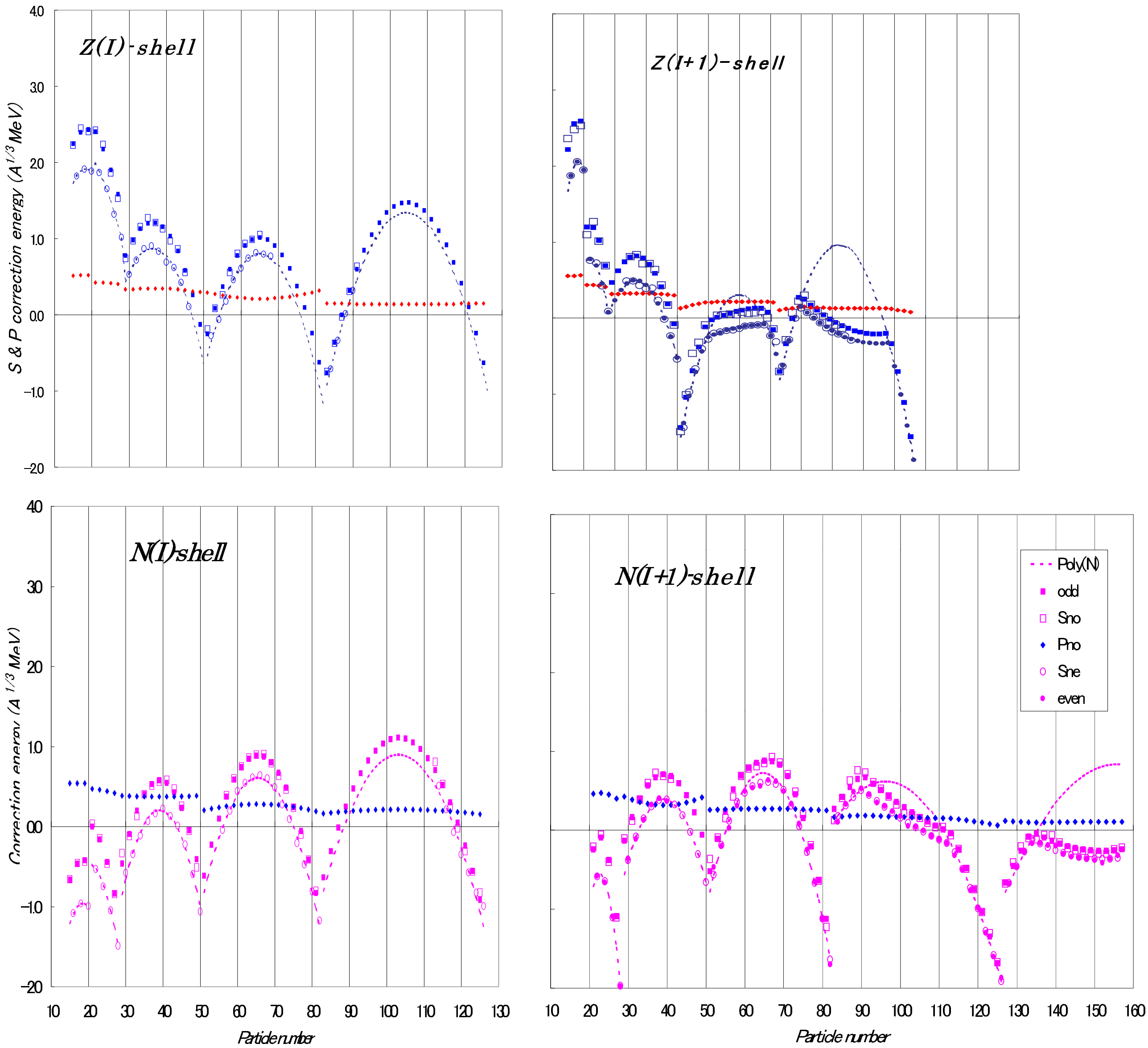

Fig.1 Shell & pairing correction energies of a new mass-formula. Poly (----) is the polynomial expressions of shell corrections without deformation effects for even-even nuclei. Symbols (□,○) represent the shell-wise averaged shell & pairing corrections simulated on the basis of RIPL2 [7] experimental mass-excess data (1995). Symbol (◆) is pairing energies fitted by using shell-pairing correlation term of Eq.(1) for odd-particle numbers. Values of mass-formula are presented by solid symbols, ■ and ●, for odd- and even-particle number, respectively.

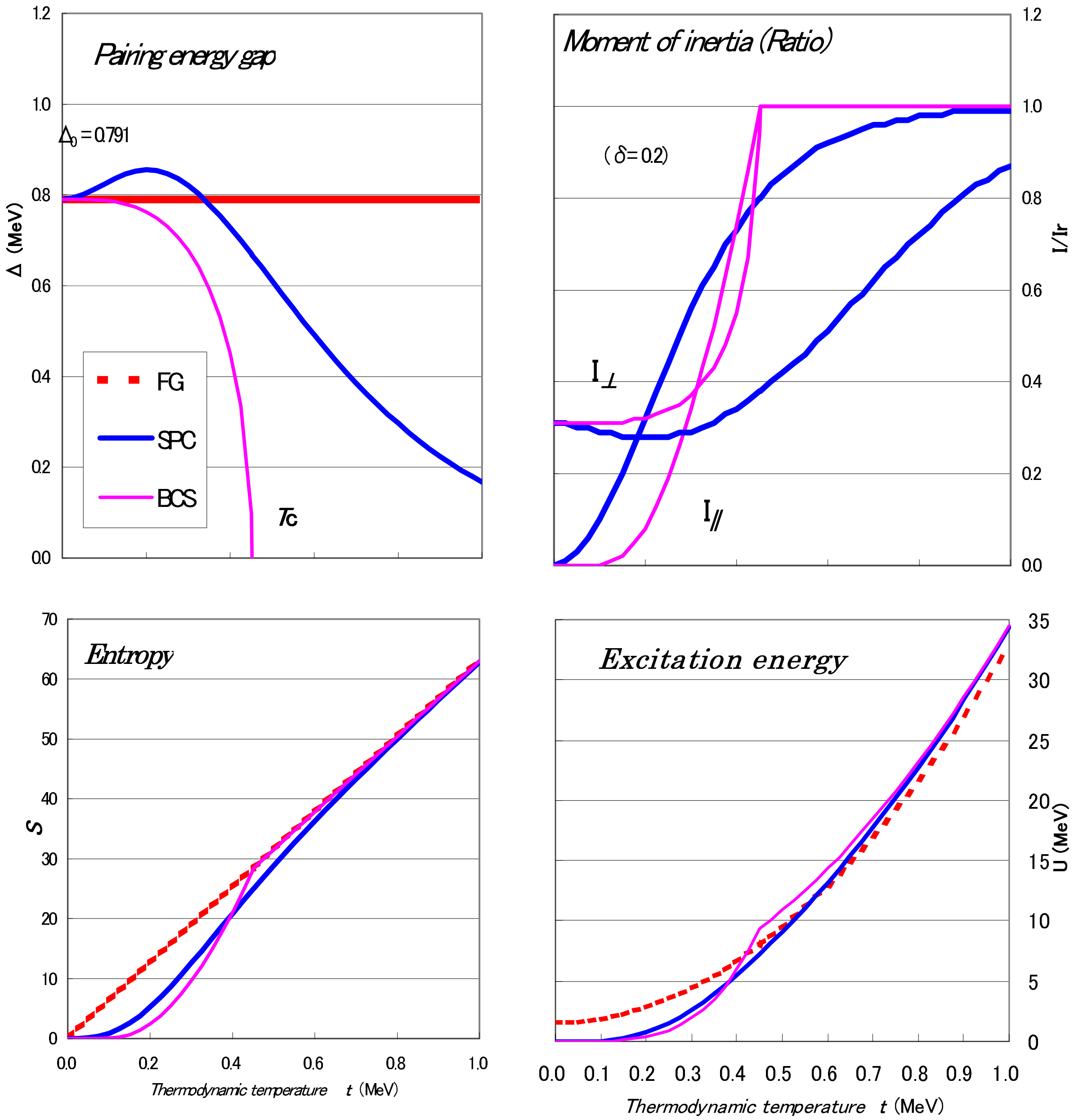

Fig.2 Temperature dependence of pairing correlations in nucleus. Comparisons of 4 kinds of thermodynamic quantities between three models, FG , BCS and BCS (present model). Those of BCS are based on analytical approximations,and the existence of a sharp phase transition point (Tc) is a special feature. For both BCS and SPC moments of inertia for two different directions, parallel and perpendicular to rotational symmetry axis are presented, the later ones do not tend to zero at the ground stste due to the existence of pairing-rotation correlations (Migdal A.B.,1959)[8]. Values of present model are based on the following Eq.(2) for a case of fx = 0 in Eq.(1) in the text and on the method of statistical thermodynamics :

$$g(\varepsilon) = g_0\{1-\cos\omega_P(\varepsilon-\lambda)\} \qquad (2)$$

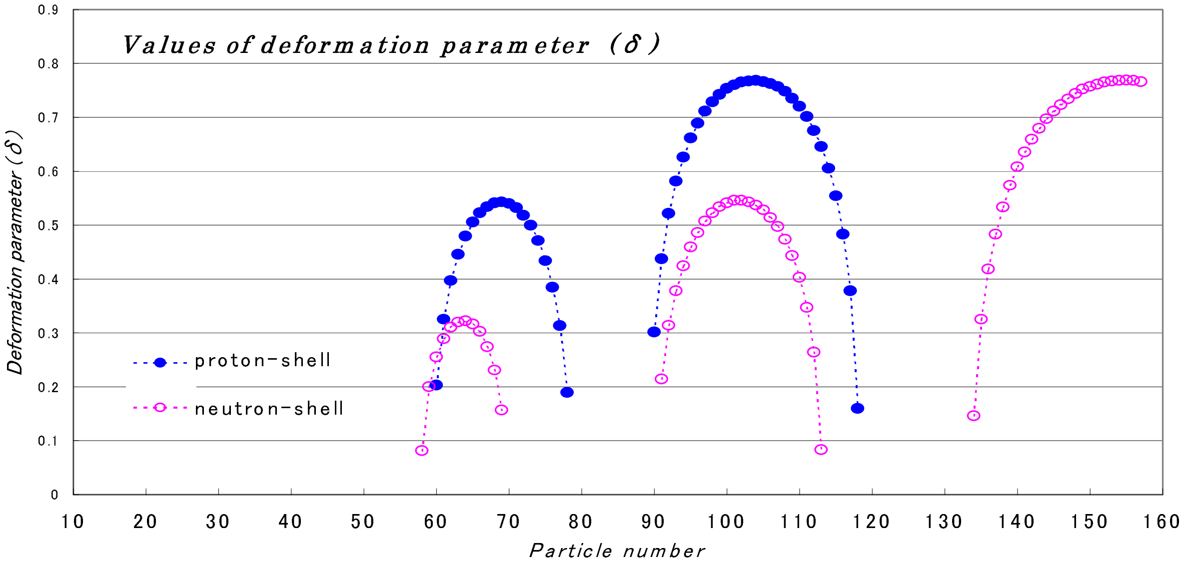

Fig.3 Values of quadrupole deformation parameter (δ).for [Z(I) — N(I+1)]-shells. Equation for determining values of δ is from Eq.(1)

$$\mathrm{Exd}(\delta,\chi) = 2g_{0}\,x\,\omega^{-2} f_x \left\{ \cos 2\pi d_i(\chi - 1/2) - \frac{1}{3} \Sigma d_i^{-2} \cos 2\pi d_i(\chi - 1/2) \right\} \quad (3)$$

$$d_1 = d_2 = (1 + \frac{1}{3}\delta), \qquad d_3 = (1 - \frac{2}{3}\delta)$$

where Exd(δ, χ) is deformation energy derived from the mass formula shown in Fig.1, χ the occupation fraction of particle (p or n) in the shell.

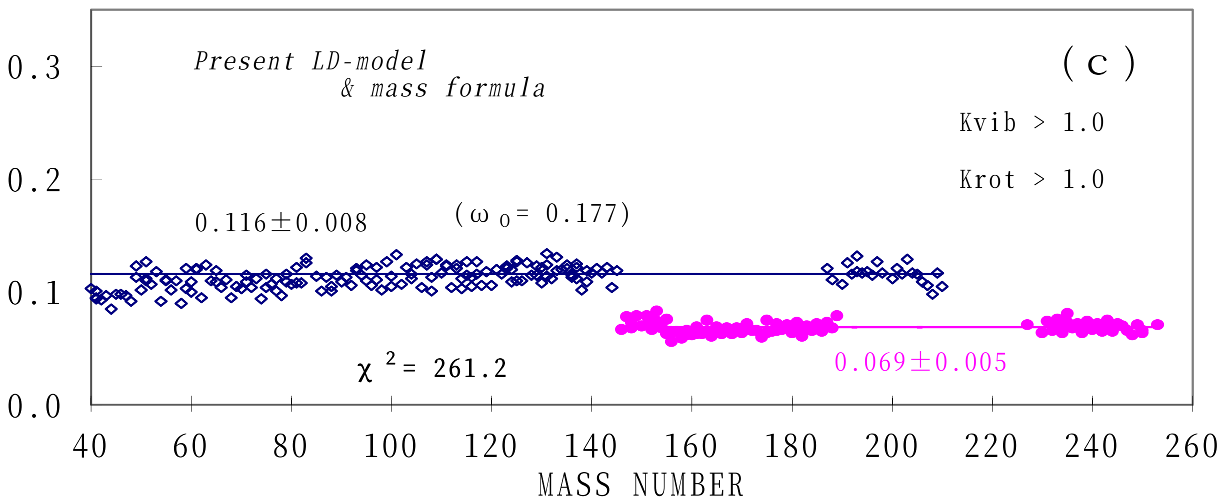

Fig.4 Systematics of asymptotic level density parameter $\alpha = a_0/A$. In the range of deformed nuclei, values of α are lowered due to the collective enhancement factors of Kvib (vibration) and mainly Krot (rotation).