Quantum recurrence in neutron resonances and a time unit in nuclei

Makio OHKUBO
SOHONRR, 1663-39,Senba-cyo, Mito-shi, Ibaraki-ken 310-0851 Japan

Abstract

Non-statistical properties of the neutron resonance level dispositions are investigated considering time behaviors of the compound nucleus. A resonance which is a quasi-stable state can be decomposed into a set of normal modes. From the requirement of the time periodic recurrence of the resonance state, frequencies of these normal mode must be commensulable (integer ratios) with each other. Therefore the excitation energy of a resonance is described as a sum of inverse integers. We tentatively adopt an expression $E_x = \mathbf{C} \sum \frac{1}{n}$, where n=integer and C=34.5MeV. Time unit in recurrence is 1.20$\times 10^{-22}$s. Possible sets of inverse integers are deduced for the resonances of light e-e nuclei.

1. Introduction

Non-statistical descriptions of the fine-structure resonances will be a developing field of the nuclear physics. For a long time more than 3-decades, non-statistical distributions of neutron resonance levels are reported by several authors[1-8]. The methods of analyses are Dij distributions (spacings between arbitrary two levels), Fourier analysis, and the compilation of levels or spacings of many levels, etc. By these analyses, dominant level spacings are found which appear more frequently than the statistical predictions. For the neutron resonance levels of light e-e target nuclei up to several hundred keV, many of the dominant spacings D_0^* can be expressed as D_{ij} =C /mn, where C=34.5 MeV and m,n are integers[8].

These features of level distributions are diametrically different from the predictions of the statistical model, which is based on the random hypothese on the highly excited states of the compound nucleus.

In order to interprete these regularities in real resonances, we have developed the "Recurrence model" of the compound nucleus[9], where time periodic behaviors of the resonance reactions are explicitely discussed.

In this article, are described the recurrence relation for normal mode ensemble, and the expansion of nuclear excitation levels into sum of inverse integers. Sets of integers for resonances of ^{32}S+n etc. are deduced.

2. Time periodicity of resonance reactions

Neutron plane wave have a form exp$i(kx - \omega t)$, where k is wave number vector, ω angular frequency, x and t are space and time coordinates. Wave packet length or coherent length of neutron is $\sim 10^{-8}$ m, measured by the neutron interferometry, or by neutron resonance width, which is suffieciently longer than the nuclear radius. Scattering of the incident plane wave by a spacially peridic scatterer induce "Bragg reflection", where kx term plays essencial role, and the diffraction patterns is the Fourier transform of the periodic lattice structures.

Similarly, scattering by time periodic scatterer induce resonances where ωt term plays essential role. At a resonance, time period of incident neutron wave $2\pi/\omega$ will be in an integer ratio to the time period τ of the compound nuclear system $(A + 1)$. Neutron wave incident on target nucleus is divided into passing component and penetrating component. The passing component passes by the target nucleus without interaction, and the penetrating one excites many degrees of freedom on the compound nucleus, and comes out with much information on the compound nucleus. The observed cross section is the resultant of these two components. At a resonance, time structures of the coming out component must be time periodic, coherent with the passing component, and constructive interference occurs in succeeding wave train. At off resonance, destructive interference occurs and make no effect except potential scattering. In a word, resonance phenomena

in energy domain inevitably relate to the recurence phenomena in time domain.

3. Recurrence relation

Eigen functions of stable descrete states are time peridic functions. This will be true for the descrete levels of nuclei with many degrees of freedom. We assume that the states of descrete levels of the compound nucleus can be approximately expanded into a set of M normal modes (M≤10), which are independent with each other and oscillating with angular frequencies $\omega_j, (j = 1, 2, .., M)$. Total Hamiltonian is a sum of ones for these normal modes.

$$H = H_1 + H_2 + ... + H_M, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The compound nuclear states $\psi(x, t)$ can be described as a direct products of these normal modes,

$$\psi(x, t) = \psi_1(x_1, t) \otimes \psi_2(x_2, t) \otimes ...\psi_M(x_M, t). \qquad \ldots\ldots (2)$$

Though the detailed structures of $\psi_j(x_j, t)$ are not known, following recurrence theorem for many oscillators system will be valid.
As j-th normal mode oscillates with a fixed frequency ω_j, the initial phase reappears every time period of $\tau_j = 2\pi/\omega_j$. Influence of the tolerable phae error of 1 rad.[9] is negligibly small in the frequency ratios. Total compound nuclear system recurs with a time period τ which is the least common multiple (LCM) of time period τ_j of each normal mode. Therefore, the time periods of these normal modes must be commensulable(integer ratios) with each other, with a time unit τ_0. Then, the total energy of the compound nucleus E_x, sum of $\hbar\omega_j$, must be written as a sum of inverse integers,

$$E_x = \hbar(\sum_{j=1}^{M} \omega_j) = 2\pi\hbar \sum_{j=1}^{M} \frac{1}{\tau_j} = (\frac{2\pi\hbar}{\tau_0}) \sum_{j=1}^{M} \frac{1}{n_j}, \qquad \ldots\ldots (3)$$

where n_j are integers and τ_0 is time unit for this resonance. This expression will be valid for energies E_x of the bound and unbound descrete levels.

4. Excitation levels and Time unit

In a previous report[8], we have shown that there are special spacings (dominant spacing) which appear frequently between two arbitrary resonances. For the resonances of 15 light e-e target nuclei up to several hundred keV, 30 dominant spacings D_0(recoil effect corrected) are found. Among these D_0, integer ratios and the least common energy(LCE) (similar to the least common multiple for two integers) are found. LCE distributions are shown in Fig.1, where peaks appear at 1406, 2027, 2655,...,9100keV. Energy ratios among these peaks indicates that they are from few common values, divided by integers. Then we tried second LCE process(LCE-2). Distributions of LCE-2 are shown in Fig.2, where peaks appear at 34.5, 39.9, 48.6, 57.6MeV, where 34.5MeV is the predominant peak. If we use peaks with circles in Fig.1, distributions of LCE-2 are shown in Fig.3, where the 34.5MeV peak remains. Though some umbiguity exists, we got a result that many of D_0 are written as D_0 = C/mn, where C=34.5MeV and m,n = integers.
Therefore, in Eq.(3), we tentatively adopt excitation energies and τ_0 expressed as

$$E_x = C \sum \frac{1}{n} \qquad (n : integer), \quad C = 34.5MeV, \qquad \ldots\ldots (4)$$

and the time unit

$$\tau_0 = (\frac{2\pi\hbar}{C}) = 1.20 \times 10^{-22} s. \qquad \ldots\ldots (5)$$

This time unit is neary equal to the reaction time measured by other experiments.

5. Expansion in sum of inverse integers

Roughly speaking, the normal modes here correspond to the excitons [10], of which number are estimated as $n_x \sim (g_0 E_x)^{1/2}$ where E_x is excitation energy in MeV, $g_0 = A/13$ the single particle level density at the Fermi energy for mass number A. For nuclei of A$\sim$30, average $n_x is \sim$ 3 at $E_x \sim$ 5MeV.

To prove the validity of Eq.(4), we tried to expand E_x in sum of C$/n_j$ less than three terms for levels of light nuclei using a computer, where $n_j \leq$ 100, with relative accuracy less than 1x 10^{-3}. Resonance parameters are taken from a book of recent eddition[11].

For example, in ^{32}S+n, the 1st 1/2+ resonance is at E_n =102.7 keV. Excitation energy is $E_x = S_n + E_n^*$= 8741.2keV, where S_n=8641.6keV is neutron separation energy and E_n^*=$E_n(A/(A+1)$ is recoil corrected neutron energy. A possible expansion for the above resonance is E_x = C[1/5 +1/30 +1/50] = 34500(38/150) =8740keV. A number set (5,30,50) is considered as a possible index of this resonance. For ten 1/2+ resonances of ^{32}S+n up to 1.66 MeV, E_n, E_n^*, E_x, possible index, LCM, E_{rec}, and δ etc. are shown in Table 1. A shematic energy levels of C/n are shown in Fig.4.

It is interesting that for considerable number of resonances of light nuclei, E_x can be written by two terms of inverse integers, E_x=C(1/m + 1/n), where m is small numbers. Moreover, a few single term levels E_x=C(1/m) are seen. For 57 resonances of ^{32}S+n below 1.66MeV, E_x of 22 resonance are C(1/4 + 1/n), and 3 resonances are C(1/5 + 1/n). Other resonances are of three terms. For 24 resonances of ^{34}S+n below 1.47MeV, E_x of 12 resonance are C(1/5 + 1/n), and 3 resonances C(1/6 + 1/n), and no resonance with C(1/4 +1/n).

6. Level spacings

From Eq.(4), level spacings D_{ij} are written as

$$D_{ij} = E_j - E_i = C(\sum_b \frac{1}{n_b} - \sum_a \frac{1}{n_a}) = C \sum \frac{n_c}{n_a n_b}. \quad ....(6)$$

This include, as a simple case, the dominant level spacings D_0= C/mn [8].

Among 10 s-wave resonances of ^{32}S+n below 1.66MeV, D_0=575keV appears three times. These spacings coincide within error of 1 keV; D_{13}=575.4keV, D_{37}=575.9keV, and D_{510}= 575.5keV,respectively. By the indexes in Table 1, these spacings are equal to C(1/20-1/30) =C(1/10-1/12) = C/60= 575 keV. Another spacing D_0=358keV appears two times; D_{37}= 358.6keV,and D_{710}= 358.2keV=C/96. The ratio 358keV/575keV is equal to 5/8. We denote the spacings 575keV as "a", 358keV as "b". Dispositions of the above resonances are described as /a/b/a-b/b/, where "/" means a real resonance. Amang these resonances, spacing /a+b/=934keV appears two times. In these level dispositions, symmetric patterns /a/b/a/ are seen with a spacing ratio 8:5:8, and /b/a-b/b/ with a ratio 5:3:5. Symmtric patterns similar to the above can be found frequently for the nuclei of wide mass region.

7. Recurrence energies

For the 1st resonance of ^{32}S+n at 102.7 keV, a possible index 5,30,50 are propotional to the time periods of the normal modes excited for this resonance. Therefore, the recurrence time period of the compound nucleus is 150τ_0 as the LCM of the indecis. We define the "Recurrence energy E_{rec} " as

$$E_{rec} = C/(LCM), \quad ....(7)$$

where C=34.5MeV. For this resonance, E_{rec}=C/150=230 keV.

There might be simple integer ratio between time period of incident neutron wave and the compound nucleus; ie. between E_{rec} and E_n^*. We tried to calculate R and δ defined below,

$$\delta = E_{rec} \times R - E_n^* \quad .... (8)$$

where R is simple integer(or half integer) which minimize | δ |. For this resonance, E_{rec}= 230 keV, E_n^* = 99.6 keV, and if we take R=1/2, $\delta = 230 \times (1/2) - 99.6 = 15.4 keV$. For ten

1/2+ resonances of ^{32}S+n up to $E_n \leq$ 1.66MeV, parameters are shown in Table 1. It is interseting that δ are 10$\sim$25keV and δ distribute around S_n - C/4 = 16.6 keV. Similar results are obtained for 47 resonances of $1/2^-$,$3/2^-$,$3/2^+$,$5/2^+$ states of ^{32}S+n.
For the resonances of ^{34}S+n, many of E_x are expressed as E_x=C(1/5+1/n), and δ distribute around S_n - C/5 = 85.8keV.

8. Discussions

From the observes neutron resonance data in light nuclei, we have reduced a common factors C on the resonance energies and τ_0 for the reaction time. Another common factors may exist in different mass region.
In order to back up the reality of C/n expansion, we searched for the case where E_x is simple terms of C/n, and have some speciality.
a) Among 43 resonances of ^{33}S+n below 548keV, Γ_n and Γ_α are extraordinaly large for 84.88keV resonance, where E_x =11499keV =C/3.
b) First excited state of ^{48}Ca is at 3832keV =C/9.
c) Neutron resonances with considerably large Γ_n have E_x with simple integer ratios to C. For example, resonance energies (recoil corrected) of first seven resonances of ^{16}O+n are (17/240)(C/6), (1/4)(C/7), (5/13)(C/7), (9/13)(C/7), (9/11)(C/7), (15/14)(C/7) and (13/10)(C/7).

Sukhoruchkin reported Δ=4.6MeV as a dominant spacing among excited states of many light nuclei, which is equal to the mass difference between $\pi^{\pm}$ and π^0. Also Δ =9m_e, where m_e is electron mass 0.511MeV.[12]. The Δ is in simple integer ratio to C. that is Δ = (2/15)C = (1/3-1/5)C.
These facts support possibility of the non-statistical description of the highly excited states of nucleus, where oscillator (exciton) energies are preseved, and simple algebra in eigen energies are valid.
Further investigations are needed to find out more clear images of the resonance compound nucleus.

References

1. S.Sukhoruchkin: Proc.Conf.Nucl.Data for Reactors, Paris,1966,Vol 1, p159, Vienna,1967 ; S.Sukhoruchkin: Proc.Int.Conf. on Statistical Properties of Nuclei (Plenum, New York 1972) p.215
2. K.Ideno, M.Ohkubo: J.Phys.Soc.Jap. 30,620(1971); K.Ideno: ibid.37,581(1974)
3. C.Coceva, F.C.Corvi,P.Giacobbe, and M.Stefanon; Proc.Int.Conf. on Statistical Properties of Nuclei(Plenum, New York 1972) p.447
4.F.N.Belyaev,S.P.Borovlev: Yad.Fiz.27.289(1978)
5.S.Sukhoruchkin: ISINN-4,Dubna 1996,ISINN-5,Dubna 1997, ISINN-6,Dubna 1998
6. G.Rohr: Low Energy Nuclear Dynamics,(World Scientific,1995),p130
7. M.Ohkubo:JAERI-Conf 98-003,p340,
8. M.Ohkubo: JAERI-Conf 2000-005,p325,
9. M.Ohkubo: Phys.Rev.C, 53.1325(1996)
10. A.Bohr and B.Mottelson;Single Particle Motion, Vol.1(Benjamin,London,1968)
11. Landolt-Börnstein New Series Vol 16/B, "Neutron Resonance Parameters", Springer(1998)
12. S.Sukhoruchkin: ISINN-7,Dubna 1998