Recurrence of the excited states of nuclei and time coherency of the de Broglie wave in ^{16}O+n resonances

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Abstract

From the requirement of the time periodicity of a (quasi)stable state, frequencies of the normal modes, which compose the state, are commensulable(integer ratios) with each other, and the excitation energies E_x are written as a sum of inverse integers. We propose an expression: $E_x = G\sum \frac{1}{n}$, where n=integers and G=34.5MeV. Recurrence time is defined as LCM(n_j) $\times$ τ_0, where τ_0= $2\pi\hbar$/G=1.20$\times$10^{-22}s. LCM vs. E_x are illustrated for all possible n_j of 2 and 3 normal modes. In ^{16}O +n resonances, integer ratios are found between the recurence frequencies of ^{17}O and the de Broglie wave frequencies of incident neutron, meaning time coherency between them. A simple branch pattern is found in ^{16}O +n resonance levels.

1. Introduction

In a compound nucleus(CN) formed by resonance reaction, it is surmised that many degrees of freedom will be excited and coupled to form chaotic mixture. Statistical properties of the observed resonances are well agreement with the predictions of random matrix theory (RMT) which is based on the random hyptheses on the CN. However, several non-statistical properties are repoted more than 3 decades, which are analysed by methods based on self-similarity in level dispositions, which are always used to decode cryptogram. The results of analyses in the neutron resonance level dispositions/spacings are described in [1-5] and references therein. Through these analyses, special level spacings (we call dominant spacings) are found which appear frequently than in ensembles of levels by RMT. Two points became clear; (a) multiple integer ratios between dominant spacings for a nucleus; (b) integer ratios among dominant spacings of different nuclei. In order to grasp more basic furcation property of resonance levels, we have made level spacing analyses on s-wave resonances of 15 even-even light target nuclei up to several hundred keV neutron energy, where simple excitations are expected because of small number of degrees of freedom excited. Thirty dominant spacings are adopted in the 15 nuclides. Among these dominant spacings of different nuclei, there are multiple integer ratios, and it became clear that many of the dominant spacings D (under recoil energy correction) are written as D= G/mn, where G=34.5 MeV, and m,n :integers. From these, it is infered that excitation energy E_x might be in a form E_x= G/k (k: integer) [6,7]. These analyses suggest following points.

(A) level furcation structures characteristic of each nucleus.

(B) a common energy constant G in many nuclei.

These features require a new view point on the compound nucleus which are diametrically different from the ordinary statistical descriptions of highly excited states.

In order to describe these regular structures in resonance levels, we are developping "Recurrence Model" of the CN [8,9], where time behaviors of CN are explicitly discussed. Assuming dynamic behaviors of the CN, mechanism of each resonance may be clarified.

In this article, are described S-matrix and the time response function, excitation energy as a sum of inverse integers, map of LCM vs. E_x, time coherence between compound nuclear recurrence and the de Broglie wave in ^{16}O+n resonances, and a special level pattern.

2. S-matrix and response function

As to the recurrence of the CN, a relation exists between S-matrix and a response function[10]. For the neutron-nucleus reaction, an S-matrix $S(E)$ is defined from which cross section $\sigma_s(E) = (\pi/k^2) \mid 1 - S(E) \mid^2$, etc.are determined. Consider incident wave $\psi^-(r,t)$ and outgoing wave $\psi^+(r,t)$ around interaction region of radius R. For s-wave, a response function $F(\tau)$ is defined as,

$$\psi^+(r,t) = \int_0^\infty d\tau F(\tau)\psi^-(r,t-\tau) \text{ ......(1).}$$

The $S(E)$ can be expressed as a Fourier integral of the response function $F(\tau)$,

$$S(E)e^{2ikR} = \int_0^{\infty} d\tau F(\tau) e^{i\frac{E\tau}{\hbar}} \text{ ....(2)},$$

where τ is previous time the response come back. For an isolated resonance at E_0, S(E) has a peak at E_0, then $F(\tau)$ must be a periodic function with a period $\tau = 2\pi\hbar/E_0$ during life time $\sim$ h/Γ. For a large resonance, $F(\tau)$ behaves like a pulse array with pulse separation τ_{rec}, like an intermittent pulses, where τ_{rec} is the recurrence time of the CN. At every recurrence time, phase of the CN recurrs almost to the initial phase, and at the time neutron density will peaks on the compound nuclear surface. If $F(\tau)$ is a non-periodic function (or τ_{rec} becomes infinity), the recurrence is not realized , and only chaotic behaviors and continuum are expected.

$F(\tau)$ can be decomposed into Fourier series with periods $\tau_j = \tau_{rec}/k_j$, where k_j are integers,(j=1,2,.. M). τ_j is written as $n_j\tau_0$, where n_j is integer and τ_0 is a unit time. τ_{rec} is the least common multiple(LCM) for ensemble (n_j) multiplied by τ_0. Frequency ratios as well as time periods among these normal modes are commensulable(in integer ratios)with each other. Frequency components ω_j of $F(\tau)$ is propotional to the inverse integers $\omega_j = (2\pi/\tau_0)/n_j$. Coherent sum of the Fourier components composes pulse array in $F(\tau)$, which describe a resonance scatterer for incident neutron wave. As to resonant reaction mechanism, neutron wave tunnels through the dynamical potential build up on nuclear surface with a time period τ_{rec} which behaves like an array of "time slit", through which interference takes place between passing and trapped components of a incident neutron wave packet. If the recurrence is coherent with the incident de Broglie wave, constractive interference takes place, and induces resonance reactions. Otherwise, destructive interference induces no reactions except for the potential scattering. An illustration of time sequence of resonance reactions is shown in Fig.1.

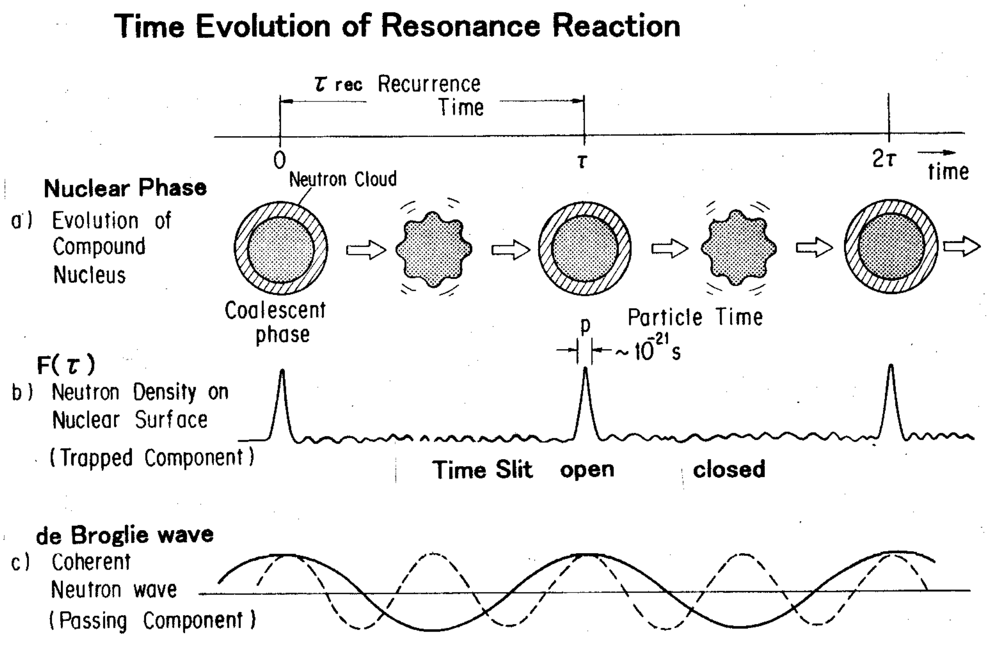

Fig.1 Time evolution of resonance reaction. a) Compound nuclear phase. b) $F(\tau)$ or Neutron density on nuclear surface or Amplitude of the initial state. Time slit open at high amplitude. c) incident de Broglie wave.

It is considered that the responce function $F(\tau)$ is under the influence of an ensemble of normal modes of which total excitation energy is $E_x = \hbar(\omega_1 + \omega_2 + .. + \omega_M)$, written as a sum of inverse integers,

$$E_x = \frac{2\pi\hbar}{\tau_0} \sum_{j=1}^{M} \frac{1}{n_j} \qquad (n_j \text{ integer}) \text{ ....(3)}$$

Compound nucleus formed by neutron resonances can be apploximately decomposed into M normal modes (M $\leq$ 10) of which frequencies are ω_j. Total Hamiltonian is a sum of these normal modes.

$$H = H_1 + H_2 + ... + H_M, \qquad \text{............... (4)}$$

The compound nuclear states $\psi(x,t)$ are described as a direct products of these normal modes,

$$\psi(x,t) = \psi_1(x_1,t) \otimes \psi_2(x_2,t) \otimes \ ... \ \psi_M(x_M,t) \ . \ ......(5)$$

Though the detailed structures of $\psi_j(x_j,t)$ are not known, the requirement of time periodicity must be fulfiled.

$$\psi_j(x,t+\tau_J) = \psi_j(x,t) \quad (j=1,2,...M) \ .... \ (6)$$

where $\tau_J = \tau_0 \times n_j$. Total recurrence is,

$$\psi(x,t+\tau_{rec}) = \psi(x,t) \ .... \ (7)$$

where $\tau_{rec} = \tau_0 \times$ LCM$(n_1, n_2, .., n_M)$.

A unit time is not necessary to be a constant for resonance to resonance. However, there are many cases where $2\pi\hbar \ /\tau_0$ =G(=34.5MeV) are common for D_0 and E_x. If G is a constant for a nucleus, level spacings D is, from Eq.(3), in a form,

$$D = G\sum(\frac{1}{m} \pm \frac{1}{n}) \ = G\frac{m'}{n'} \ \text{(m',n' : integers) ...(8).}$$

The above discussion can be applied to general (quasi) stable states because of time periodicity of the states. An example is ground state rotational band of ^{50}Cr, where E_x are written in a form E_x= G'/n, where n=5,6,8,12, and G'=(11/10)G [9].

3. Ensembles of the minimum LCM

For a (quasi) stable state of nucleus, there are many ensembles of normal modes possible to be excited which satisfy the energy relation in Eq.(3). However, we think that only a few ensembles of the minimum recurrence time or smallest LCM will be excited strongly. This mean that ensembles of n_j (j=1,2,...,M) with simple integer ratios will be realized. This will be analogous to the Fermat's principle in optics, or Hamilton's principle in mechanics ; the real path minimize the transit time from a point A to another point B. In our case of recurrence, phase point start from A to reach B, which is eqal to A.

Using Eq.(3), we have searched for possible ensembles and their LCM for two and three normal modes where E_x is defined with G=34.5MeV.

2-normal modes $\quad E_x = G(\frac{1}{n_1} + \frac{1}{n_2}) \quad$ .... **(9)**

3-normal modes $\quad E_x = G(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}) \quad$ .... **(10)**

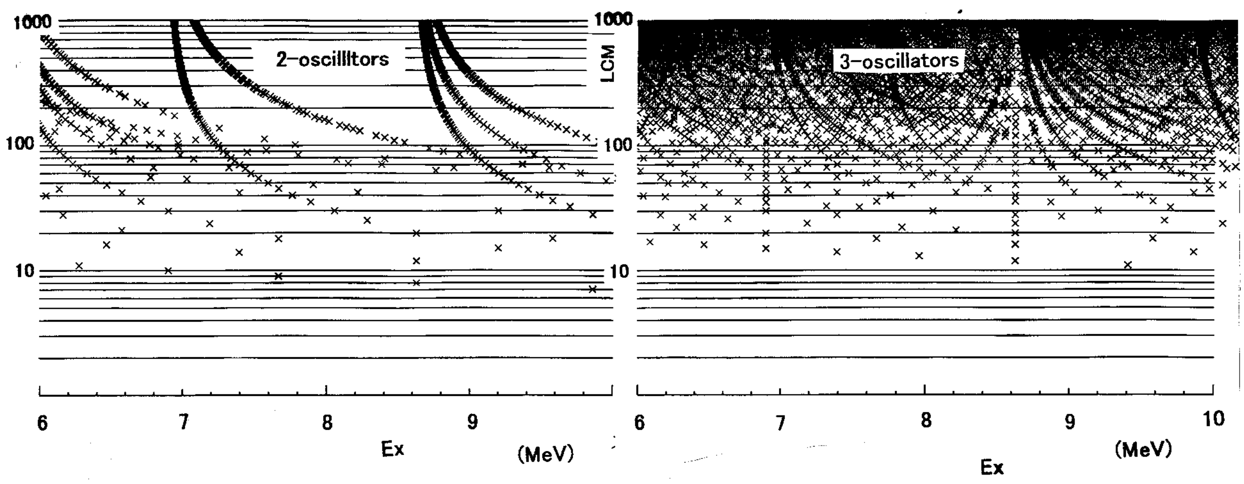

Fig.2 LCM vs. E_x for LCMle1000 and E_x from 6 to 10MeV. a) 2-normal modes b) 3-normal modes.

As shown in Fig.2, there are many special E_x where LCM are small. At E_x= G/n, LCM make dips, and around them LCM become large. Among the points several bands are found , with simple meaning on the LCM. It is interesting that around E_x= G/n accompaning bands are found having asymptotes at E_x= G/n (n= 2,3,4,...). For 2-normal modes, a common band with the same curvature are found, with E_x and LCM written as

E_x = G(1/n + 1/nm) (n=2,3,4,...; m=1,2,3...) ....(11)

LCM = nm. ....(12)

For n=3 band, the elements of the common band are at E_x/G = 2/3, 3/6, 4/9, 5/12, 6/15, 7/18, 8/21,..., (k+1)/ 3k :(k=1,2,3,..) . Also for n=5 band, E_x/G = 2/5, 3/10, 4/15, 5/20, 6/25, 7/30, 8/35,..., (k+1)/5k :(k=1,2,3,...) . Between n=3 and n=5 bands, a spacing of (1/3 - 1/5)G =4.6MeV

appears many times. This will be related to the results of Sukhoruchkin[5] where level spacings D_{ij} distibutions of real nuclei show a peak atΔ= 4.6MeV for A$\leq$50 nuclei. Similarly, following spacings will be enhanced in level spacing distributions: between band 2 and 3, =5.75MeV; between 2 and 4,=8.625 MeV; between 2 and 5, =10.35MeV; between 3 and 4, =2.875MeV; between 3 and 6,= 5.75MeV,..so on. For 3 normal modes, possible points of small LCM increase, and the similar bands as above are seen.

4. Time Coherency of the de Broglie wave with the Recurrence in ^{16}O+n resonances

By an analogy of the quantum beat in laser physics, it is considered that recurrece frequency of a compound nucleus might be commensulable to the de Broglie wave frequcncy $\omega_k = E_k^*/\hbar$ of the incident neutron wave. We have investigated integer ratios of E_x/E_n^* for the resonances of light nuclei, where E_n^* is recoil corrected neutron energy. For many of the resonances of ^{16}O+n, simple integer ratios are found; E_x/E_n^* =n/m within an accuracy less than $\sim 2\times10^{-3}$. Newly evaluated data of 36 levels below $E_x \leq$10 MeV [11] are used. Results are shown in Table 1.
For example, a sharp resonance at E_n=1651.4keV(E_n^*=1553.3keV using neutron mass=1.0092, and ^{16}O=15.995), E_x=5696.7 keV , the ratio E_x/E_n^* = 11/3. As E_x=(33/200)G =(1/8 + 1/25)G and LCM=200, the recurrence energy is E_{rec} = G/200 = 172.6keV. Neutron energy is E_n^*=(9/200)G = 9E_{rec}, and neutron separation energy is S_n 4143.3keV = 24E_{rec}= (3/25)G. Difference δ between E_n^*=1553.3keV and 9E_{rec} is, $\delta=E_n^*$ - 9E_{rec} = -0.3 keV or ratio δ/E_{rec}= 2$\times10^{-3}$. This means that in a recurrence time of ^{17}O 200$\times\tau_0$= 2.40$\times10^{-20}$ s, incident de Broglie wave oscillate exactly 9 cycles. This coherency hold during life time (h/Γ, Γ ~4.1keV) of this resonance $\sim 1\times10^{-18}$s. This supports the resonance reaction mechnism described in section 2. In the 36 levels of ^{16}O+n resonances, we found 15 cases of integer ratios with m$\leq$11 and n$\leq$20, as shown in Table 1.The other resonances are excluded because the ratios E_x/E_n^* are between large integers.

Table 1 Ratios of de Broglie frequencies to the Recurrence frequencies in ^{16}O+n

j	jπ	E_x (keV)	E_n (keV)	E_x/E_n^*	E_x/G†	LCM	E_{rec} (keV)	R	δ (keV)	δ/E_{rec}
1	7/2-	5696.7	1651.4	11/3	33/200	200	172.6	9	0.2	0.001
2	5/2-	5732.3	1689.1	18/5	18/108	108	318.4	5	-2.6	-0.008
3	3/2+	5868.7	1834.1	17/5	17/100	100	345.2	5	-0.6	-0.002
4	1/2-	5932.0	1901.4	10/3	10/58	58	593.2	3	7.0	0.012
5	1/2+	6380.2	2377.9	20/7	20/108	108	319.0	7	2.8	0.009
6	5/2-	7164.6	3211.7	19/8	19/91	91	377.1	8	3.2	0.008
7	3/2+	7239.1	3291	7/3	7/33	33	1034.1	3	-4.7	-0.005
8	5/2+	7378.2	3438.8	16/7	16/75	75	461.1	7	4.8	0.010
9	3/2-	7446.9	3511.9	9/4	9/42	42	827.4	4	-4.3	-0.005
10	7/2-	7686.9	3767	13/6	13/58	58	591.3	6	-2.9	-0.005
11	7/2-	8963.2	5123.7	13/7	13/50	503	689.5	7	-4.2	-0.006
12	5/2+	9194.1	5369.3	20/11	20/75	75	459.7	11	-3.9	-0.008
13	5/2-	9479.5	5672.6	16/9	16/58	58	592.5	9	2.5	0.004
14	7/2+	9710.9	5918.6	7/4	7/25	25	1387.3	4	11.7	0.008
15	9/2+	9859.1	6076.2	19/11	19/66	66	518.9	11	5.0	0.010

$\delta = E_n^*$ - R E_{rec} S_n=4143.3keV   † G varies within ±1% from 34500keV.

For two sharp resonances of No.8 and 12 in Table 1, they are same LCM= 75 and same Jπ=5/2+. Energy relations are shown in Fig.3. For No.8 resonance, E_x=7378.2keV = (1/5 + 1/75)G =(3/25 + 7/75)G with LCM=75, where (3/25)G is neutron separation energy. For No.12 resonance, E_x=9194.1keV = (20/75)G= (1/5 + 1/15)G = (3/25 + 11/75)G with LCM=75.
We would like to stress that the above analyses give evidence that the model of the compound nucleus composed of normal modes is really valid, with energies E_x= G$\sum\frac{1}{n_j}$, τ= LCM(n_j)$\times\tau_0$ with almost correct value G=34.5MeV.

5. Level Patterns in ^{16}O+n resonances

In D_{ij} distributions of ^{16}O+n resonances, shown in Fig.4, remarkable peaks appear at D_{ij}(D_{ij}^*) = 550(517.6), 1609(1515), 1930(1816) keV, which are adopted as dominant spacings. The 1930keV peak corresponds to the spacing between the two levels No.8 and 12 in Table 1. We noticed that the 1930keV spacings(we call a) frequently neighbor the another spacing 550 keV (we call b) like a branch. Level patterns of outside branch /b/a/b/ and inside branch /b/a-2b/b/ are searched, where "/" means a real resonance level. The full patterns /b/b/a-2b/b/b/ , shown in Fig.5, and partial ones where some levels except /a/ disappear, are recorded. In 37 levels in the region $E_n \leq$6207 keV, there are eight /a/ with an error a=1930±10 keV, and b = 550±10keV. For clarity, two levels at both sides of /a/ we call stem levels.

Probability of appearance of the pattern is estimated assuming all the levels are disposed homogeneously without any correlation. In observed 37 resonance levels in the region, two pairs of closedly separated levels(3 and 9 keV) exist, and we regard as two single levels. Then, the average level spacing is 177 keV, and the average number of levels in 20keV channel width is 0.113. For 6 times appearance of stem levels /a/, 14 branch levels are placed at 550±10keV separation from stem levels, where maximum possible levels is 24. Probability of of appearance P of such case are estimated by binomial distribution, as, P= ${}_{24}C_{14}\lambda^{14}(1-\lambda)^{10} \sim 1.7\times10^{-7}$, where $\lambda = 0.129$ is probability of appearance of a level in a channel of 20keV width. As the branch length b is arbitrary, number of channels n =(965/20 $\sim$ 50) must be multiplied. Therefore the expected number of set is n× P$\sim$0.9×10^{-5}, which is suficiently small to deny the assumption of random (or RMT) distribution of the resonance levels. It is stressed that several patterns different from that in Fig.5 will be simultaneously found like another patterns in periodic lattice. Therefore, we can say that the resonance levels of ^{16}O+n dispose with fairly simple structures a facet of which can be discovered by rather crude methods as above. Similar structures are found in nuclear levels of wide mass and energy region.

Average numbers of normal modes M excited in ^{16}O+n are estimated to be $\sim$ 2, by a formula M=1+ln(E_x/D) /ln(2π) [8], where D is for same Jπ levels. For two 5/2+ levels of No.8 and 12 in Table 1, a simple configulation will be expected with recurrence energy 460keV.
The time dependent descriptions of resonance reactions and non-statistical description of fine structure resonances will be a realm of nuclear research in 21 century, related to the multiphonon excitation, fine structures in giant resonances.

References

[1] S.Sukhoruchkin: *Proc.Conf.Nucl.Data for Reactors, Paris,1966*,Vol 1, p159, Vienna,1967 ; S.Sukhoruchkin: *Proc. Int. Conf. on Statistical Properties of Nuclei* (Plenum, New York 1972) p.215

[2] K.Ideno, M.Ohkubo: "Nonrandom distributions of neutron resonance levels" *J.Phys.Soc.Jap.* 30,620(1971); K.Ideno:"Nonstatistical behaviours of the level spacing distributions in neutron resonances" ibid. 37,581(1974)

[3] C.Coceva, F.C.Corvi, P.Giacobbe, and M.Stefanon; *Proc. Int. Conf. on Statistical Properties of Nuclei* (Plenum, New York 1972) p.447

[4] F.N.Belyaev,S.P.Borovlev: *Yad.Fiz.*27.289(1978), transl.*Sov.J.Nucl.Phys.*27(2),157(1978)

[5] S.Sukhoruchkin: "Shell stabilization of excitations in odd odd nuclei"ISINN-4, E3-96-336 Dubna 1996, ISINN-5, Dubna 1997, ISINN-6, Dubna 1998, ISINN-7,Dubna 1999, ISINN-8,Dubna

[6] M.Ohkubo: "Regular level structures in s-wave neutron resonances in light or magic e-e nuclei up to several hundred keV" *INDC(JPN)* -185/U ,JAERI-Conf 2000-005, p325,

[7] M.Ohkubo:"Quantum recurrence in neutron resonances and a unit time in nuclei" *INDC(JPN)* -188/U, JAERI-Conf 2001-006, p300

[8] M.Ohkubo:"Recurrence of the compound nucleus in neutron resonance reactions" *Phys.Rev.C,* 53,1325(1996)

[9] M.Ohkubo;" Non-statistical level disposition analysis in neutron resonances" *J.Nucl.Sci.Technol. Supplement 2, Proc. Int.Conf. ND2001* p508 (2002).

[10] A.G. Sitenko: *Lecture in Scattering Theory*(Pergamon, New York 1971)

[11] R.O.Sayer,L.C.Leal,L.M.Larson,R.R.Spencer,and R.Q.Wright: "R-Matrix Evaluation of ^{16}O Neutron Cross Sections up to 6.3 MeV" *J.Nucl.Sci.Technol. Supplement 2, Proc. Int.Conf. ND2001* p88, (2002).

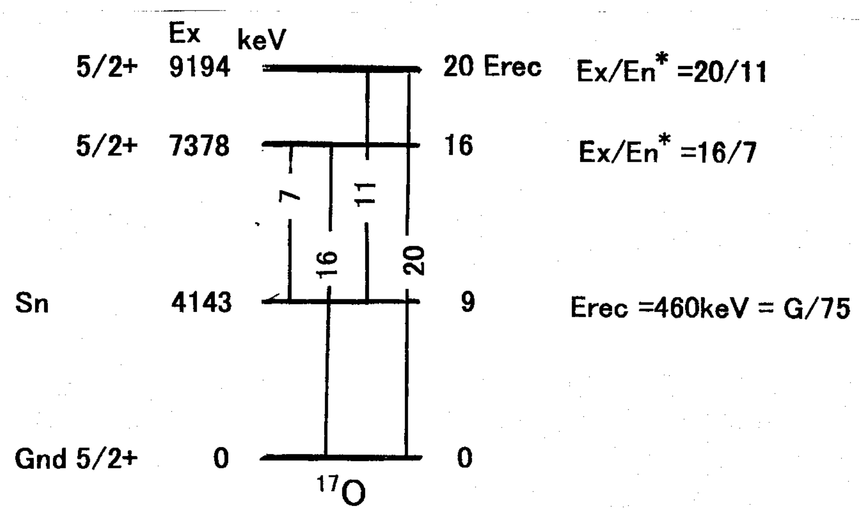

Fig.3 Levels scheme of 5/2+ with E_{rec}= 460keV.

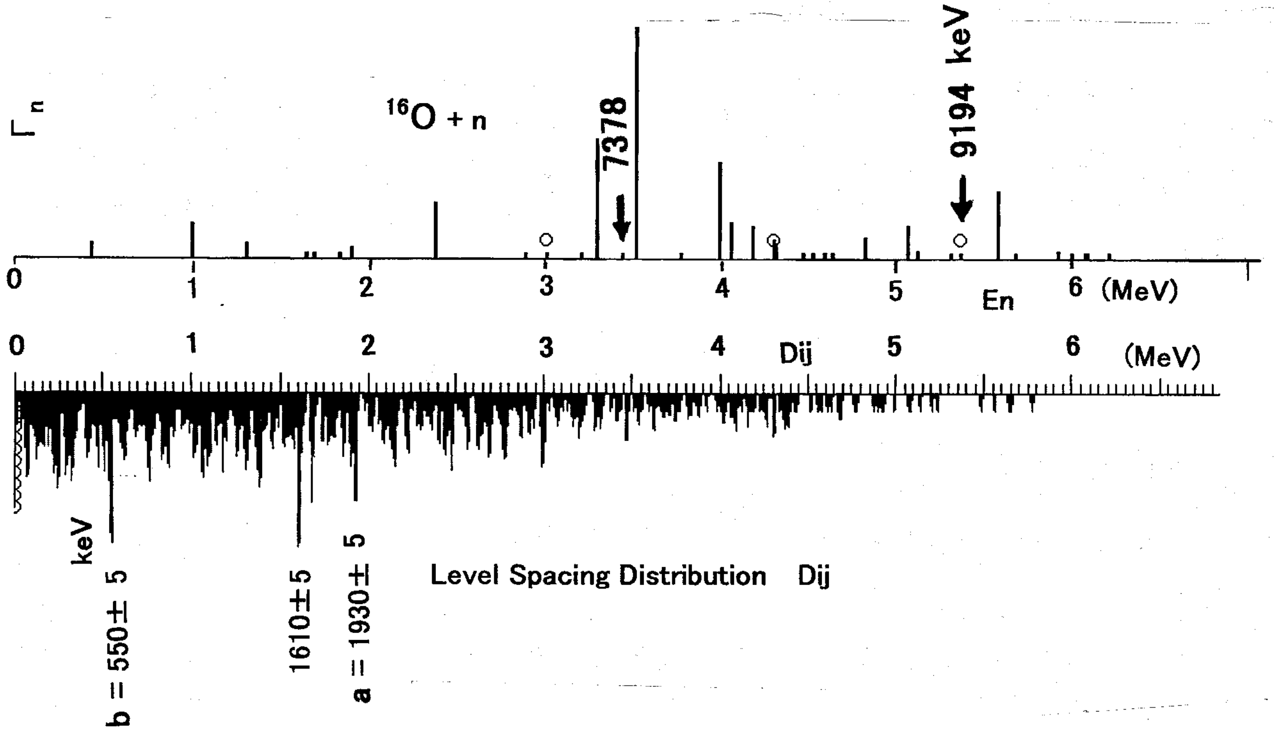

Fig.4 upper part: Neutron widths vs neutron energy for ^{16}O+n resonances. lower part: Level spacing distribution for ^{16}O+n resonances, where correlation of level spacing a=1930 keV and b=550keV are discussed in text. Spacing a=1930keV (a*= 1816keV) correspond to the difference between 5/2+ levels of 7378 and 9194 keV shown in Fig.3.

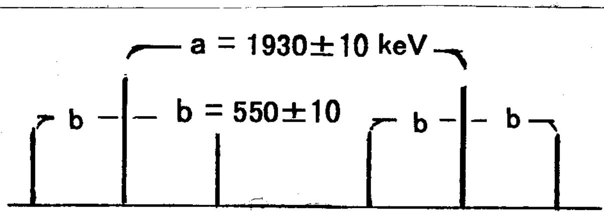

Fig.5 Level pattern searched for : stem levels a=1930keV and branch levels b free parameter. For 6 times appearance of stem levels, 14 levels are situated on branch positions at b=550keV.