Neutron Optical Potentials in Unstable Nuclei and the Equation of State of Asymmetric Nuclear Matter

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Neutron single particle potential is one of the basic macroscopic properties to describe structure and reactions of nuclei in nuclear reactors and in the universe. However, the potential is quite uncertain for unstable nuclei primarily because the equation of state (EOS) of asymmetric nuclear matter is not known well. The present authors studied systematically the empirical EOS of asymmetric nuclear matter using a macroscopic nuclear model; about two hundred EOS's having empirically allowed values of *L* (symmetry energy density derivative coefficient) and K_0 (incompressibility) were obtained from the fittings to masses and radii of stable nuclei. It was suggested that the *L* value could be determined from global (Z, A) dependence of nuclear radii. In the present study, the single particle potential is examined assuming kinetic energies of non-interacting Fermi gases. The potential in a nucleus can be calculated easily, once the density distribution is solved using the effective nuclear interaction (EOS). Neutron and proton single particle potentials are calculated systematically for ^{80}Ni using the two hundred EOS's. It is found that the neutron-proton potential difference has clear and appreciable *L* dependence, while the potential for each species does not show such simple dependence on *L*.

1. Introduction

In the past, the present authors independently studied extremely neutron-rich nuclei in neutron star matter and supernova matter. The key for the description is found the equation of state (EOS) of asymmetric nuclear matter, which determines the bulk property of neutron rich nuclei. The empirical EOS of symmetric nuclear matter is determined well from stable nuclei. While the asymmetric matter EOS should be, in principle, determined from neutron rich nuclei, it has not been possible because experiments using sufficiently neutron rich nuclei has been prohibitingly difficult.

Now the progress of experimental technique is opening the door to experimental study of neutron rich nuclei, the authors have studied possibility of empirical determination of the EOS using a macroscopic nuclear model. Firstly, about 200 empirically allowed EOS's are constructed systematically by fitting masses and radii of stable nuclei. Secondly, properties of neutron rich nuclei are calculated using the EOS's. It is found that masses and radii of unstable nuclei has appreciable dependence on the EOS. The key parameter is found *L* (symmetry energy density derivative coefficient), which can not be determined from stable nuclei.

Now we turn to the EOS dependence of optical potential parameters. In the present study, the potential depth of the neutron and proton single particle potentials is studied using the 200 EOS's.

2. Key EOS parameters

In this paper, we deal with the energy per nucleon of nuclear matter, *w*, as a function of nucleon number density *n* and proton fraction *x*. The empirical EOS is characterized by a limited number of parameters. It is useful to consider Taylor expansion of the energy per nucleon, which is given by

$$w(n,x) \approx w_0 + \frac{K_0}{18n_0^2}(n-n_0)^2 + (1-2x)^2\left[S_0 + \frac{L}{3n_0}(n-n_0)\right]. \tag{1}$$

Here, n_0 and w_0 are saturation density and energy of symmetric nuclear matter, respectively. They are well determined from stable nuclei. The incompressibility of symmetric matter, K_0, should also be determined from stable nuclei but its values is somewhat uncertain even at present. The parameters of interest here are S_0, the symmetry energy, and the symmetry energy density derivative coefficient, L. In terms of the density dependent symmetry energy $S(n)$, these parameters are defined as $S_0=S(n_0)$ and

$$L = 3n_0 dS/dn\big|_{n=n_0}. \tag{2}$$

The saturation point (density and energy) of asymmetric nuclear matter mainly determines nuclear masses and radii of neutron rich nuclei. A useful and intuitive EOS parameter to characterize the saturation point of asymmetric nuclear matter is the slope of the saturation line (the line joining the saturation points) at $n=n_0$, which is given by

$$y = -\frac{S_0K_0}{3Ln_0} \tag{3}$$

In the followings, this parameter, y, will be used to empirically constrain the symmetry energy.

3. Empirically allowed EOS of asymmetric nuclear matter

In this study, a macroscopic nuclear model is used to derive the empirical EOS of asymmetric nuclear matter because the EOS is nothing but a macroscopic property of nuclei. We begin with a model for the bulk energy density of uniform nuclear matter. The energy density, $w(n,x)\times n$ is written as the sum of kinetic and potential energy densities as a function of the neutron density n_n and the proton density n_p; and total nucleon density $n=n_\mathrm{n}+n_\mathrm{p}$;

$$\varepsilon_0(n_n, n_p) = \frac{3}{5}\left(\frac{3\pi^2}{2}\right)^{2/3}\left[\frac{\hbar^2}{2m_n}n_n^{5/3} + \frac{\hbar^2}{2m_p}n_p^{5/3}\right] + \left[1-(1-2x)^2\right]v_s(n) + (1-2x)^2 v_n(n) \quad . \tag{4}$$

Here, $n=n_\mathrm{n}+n_\mathrm{p}$ and $x=n_\mathrm{p}/n$. The first term in Eq. (4) is the Fermi kinetic energies of neutrons and protons. The functions $v_\mathrm{s}(n)$ and $v_\mathrm{n}(n)$ denote potential energy densities for $x=1/2$ (symmetric nuclear matter) and $x=0$ (pure neutron matter), respectively.

$$v_s(n) = a_1n^2 + \frac{a_2n^3}{1+a_3n}, \qquad v_n(n) = b_1n^2 + \frac{b_2n^3}{1+b_3n}. \tag{5}$$

These functions have desirable limiting behavior; two body interactions are dominant at low densities while, at high densities, the sound speed does not exceed the speed of light thanks to the denominators. The present model (Eqs. (4) and (5)) is flexible enough to fit various effective interactions of contemporary use such as Skyrme type interactions in the non-relativistic theory and Lagrangeans of relativistic mean field theory.

In the Thomas-Fermi approximation, the binding energy of a nucleus is written, using the local neutron (proton) density $n_\mathrm{n}(r)$ ($n_\mathrm{p}(r)$), as

$$-B = \int d\boldsymbol{r}\, \varepsilon_0(n_n, n_p) + \int d\boldsymbol{r}\, F_0\left|\nabla n(r)\right|^2 + \frac{e^2}{2}\int d\boldsymbol{r}\int d\boldsymbol{r}'\, \frac{n_p(\boldsymbol{r})\, n_p(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}, \tag{6}$$

with $n=n_\mathrm{n}+n_\mathrm{p}$. Here $\varepsilon_0(n_\mathrm{n}, n_\mathrm{p})$ in the first term in Eq. (6) denotes the energy density, namely the EOS. The first term in Eq. (6) is the bulk energy, the second term is the gradient energy with an adjustable constant F_0, and the third term is the Coulomb energy. In this paper, we assume, for simplicity, that the nucleon distributions $n_i(r)$ (i=n,p) are parametrized as

$$n_i(r) = \begin{cases} n_i^{in}\left[1 - \left(\frac{r}{R_i}\right)^{t_i}\right] & r \leq R_i \\ 0 & r > R_i \end{cases} \quad . \qquad (7)$$

Here, R_i and t_i are the radius and surface diffuseness parameters, respectively with n_i^{in} being the central density.

For a given nucleus, the binding energy B is calculated by minimizing the Eq. (6) with respect to the variation of R_i and t_i. Consequently, the nuclear mass and radius are functions of interaction parameters, a_1-a_3 and b_1-b_3 in Eq. (5), and F_0 in Eq. (6). The six parameters, a_1-a_3 and b_1-b_3 directly determine the EOS in the present model while F_0 represents finite range effects of nuclear interactions.

The values of the interaction parameters are determined empirically to fit masses and radii of stable nuclei. The fitting procedure together with input data for the fitting are described in Ref. [1] and [2]. In the present study, the value of b_3 is fixed as a typical value $b_3 \approx 1.6$. This particular choice does not affect the conclusion of the present study.

It is noted that various interaction parameter sets can reproduce the masses and radii of stable nuclei almost equally because the neutron-proton asymmetry is not large enough in stable nuclei to determine the symmetry energy. To generate empirically allowed EOS systematically, the fittings were performed for fixed values of empirically allowed values of (y, K_0). The obtained parameter sets cover essentially all possible EOS's as shown in Fig. 1.

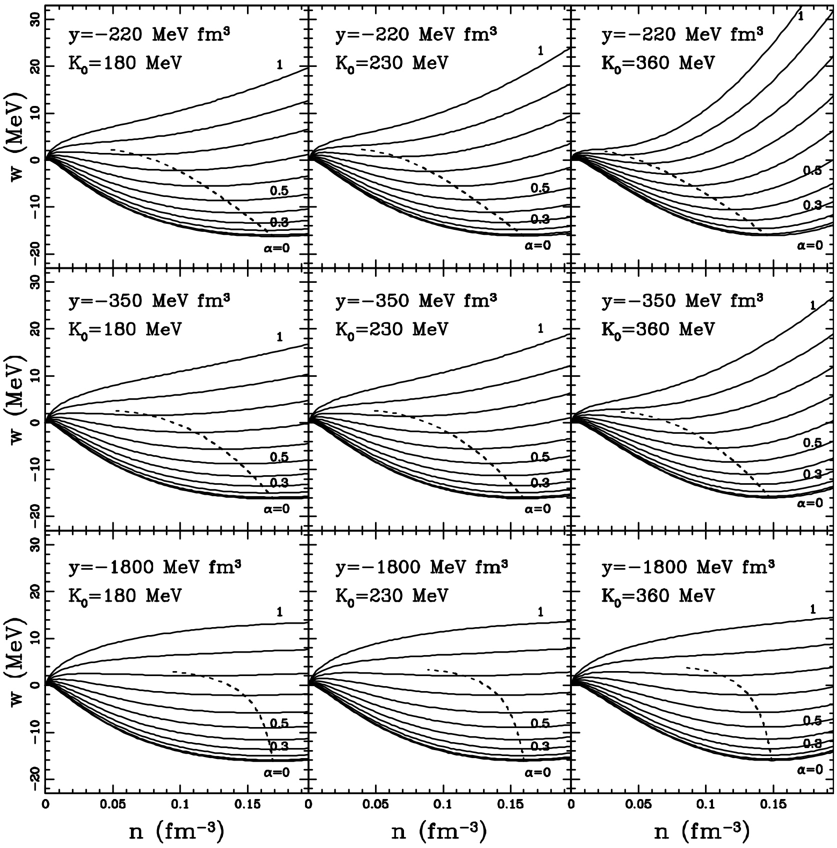


Fig. 1. Empirically allowed EOS's obtained from the fitting to stable nuclei.

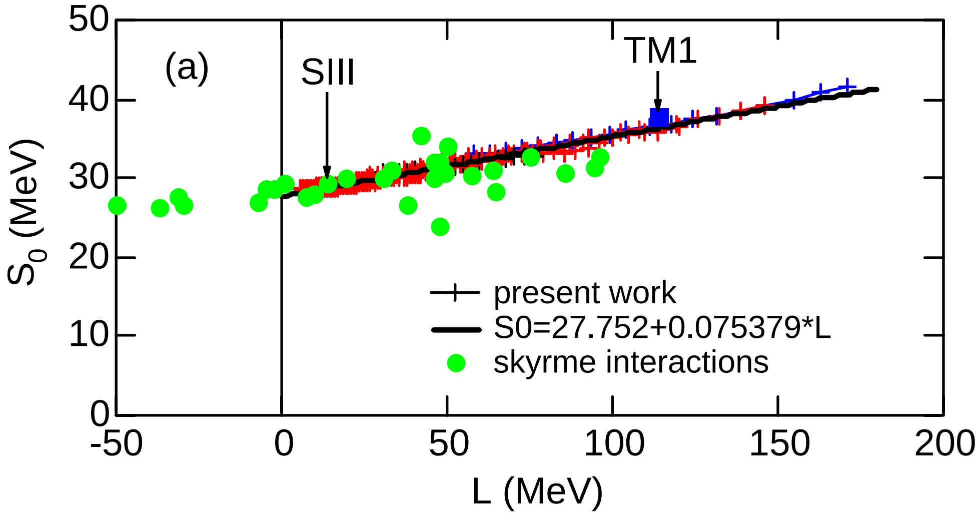


Fig. 2. Strong correlation between S_0 and L.

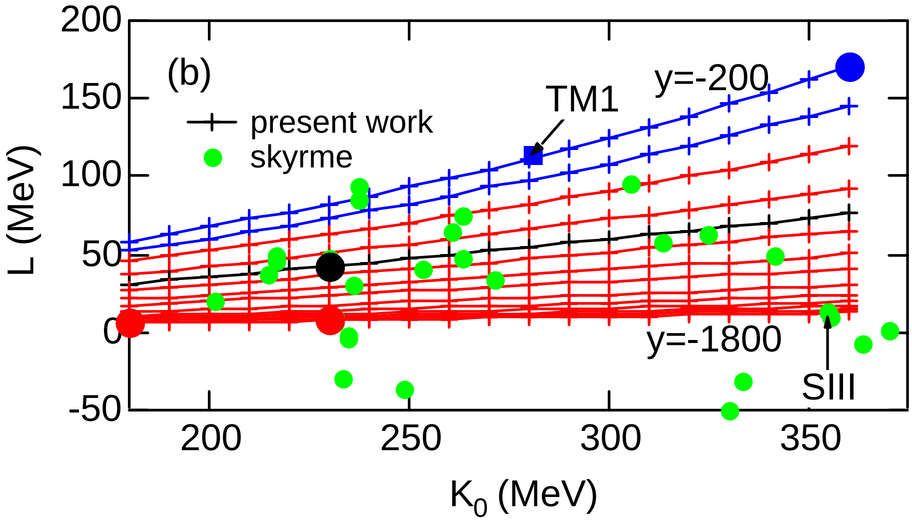


Fig. 3. Empirically allowed region of (K_0, L).

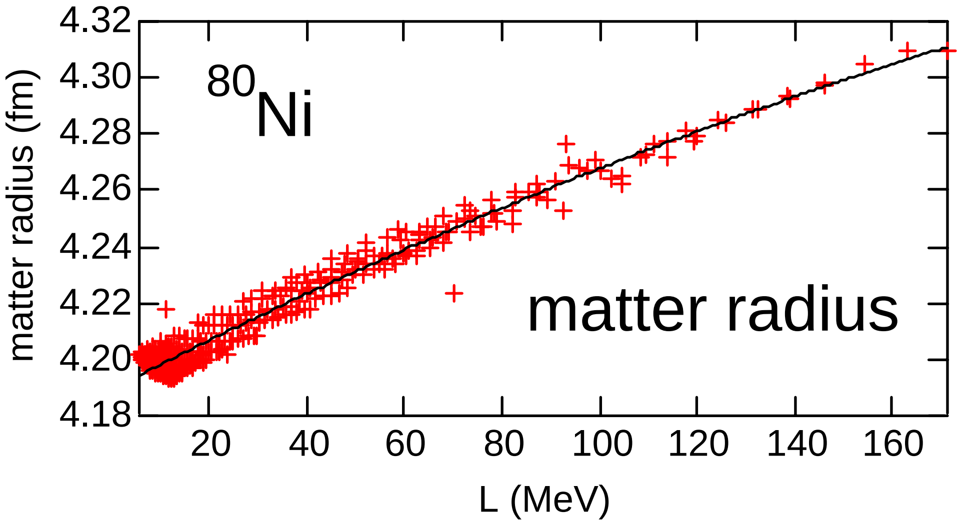


Fig. 4. Matter radius of ^{80}Ni as a function of L.

From this systematic study, it is found, for empirically allowed EOS's, that S_0 and L are strongly correlated as shown in Fig. 2. It is also found that there is an empirical allowed region of (K_0, L) shown in Fig. 3.

In Ref. [2], it is concluded that nuclear radii of neutron rich nuclei strongly depend on L as shown in Fig. 4. This feature together with the S_0-L correlation opens the door to determine the asymmetric matter EOS from measurements of nuclear radii in laboratories.

4. Single particle potential of neutron-rich nuclei

It is interesting to investigate neutron and proton single particle potentials of unstable nuclei from the viewpoints of both nuclear structure and reaction studies. As a key potential parameter, we focus on the depth of the neutron (proton) single particle potential. The single particle potentials are given by

$$U_n = \frac{\partial v(n_n, n_p)}{\partial n_n}, \quad U_p = \frac{\partial v(n_n, n_p)}{\partial n_p} + (Coulomb), \tag{8}$$

where $v(n_n, n_p)$ is the potential energy density given by

$$v(n_n, n_p) = \varepsilon_0(n_n, n_p) - (kinetic\ part) = \left[1 - (1-2x)^2\right] v_s(n) + (1-2x)^2 v_n(n) . \qquad (9)$$

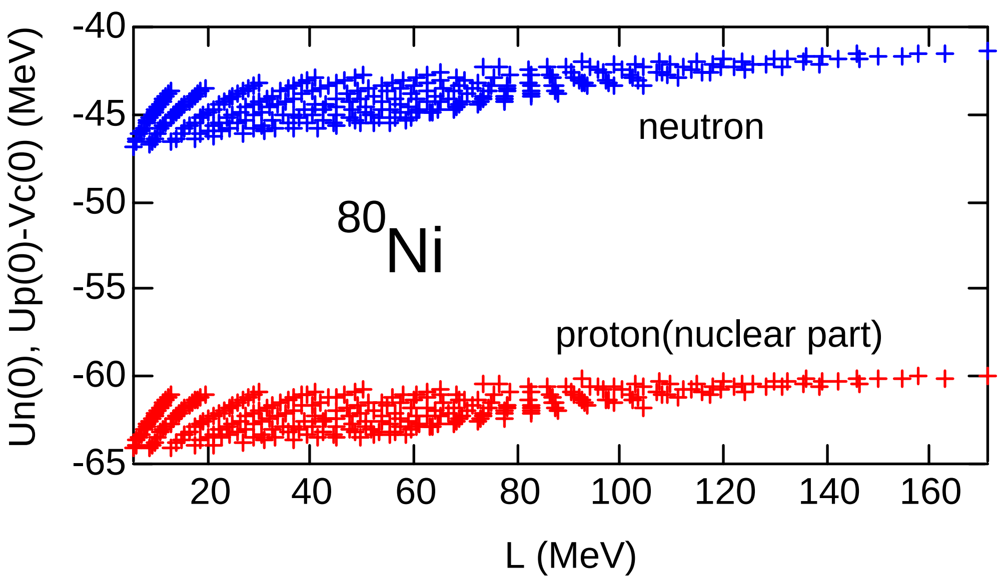
The potential depth is given by the U_n (or U_p) value at r=0. In the present model (Eqs.(4) and (9)), the effective nucleon mass is taken to be one.

In Fig. 5, the potential depth is shown for neutrons and protons as well as its difference between neutrons and protons. For proton potential U_p, the Coulomb potential is subtracted to examine effects which directly reflect nuclear interactions. It is seen that the neutron-proton potential difference depends obviously on L while such simple L dependence is not seen for the neutron (or proton) potential depth. The L dependence of the potential difference is only 1 MeV, which is relatively small but appreciable compared with its absolute value (about 18 MeV).

5. Summary

In the Thomas-Fermi model, single particle potential can be directly calculated from the EOS, once nuclear distributions are solved using an effective nuclear interaction. The present study shows, similarly to nuclear masses and radii, the neutron-proton potential difference has clear and appreciable L dependence. The L dependence reflects that fact that neutron and proton densities at the center are sensitive to L. Hence it seems interesting to examine this feature in relation to nuclear radii and formation of neutron skins of neutron rich nuclei.

[1] K.Oyamatsu and K.Iida, Proc. The fourth sympo. on Science of Hadrons under Extreme Conditions, JAERI-CONF 2002-011, pp.36-49 (in Japanese).
[2] K. Oyamatsu and K.Iida, nucl-th/0204033.



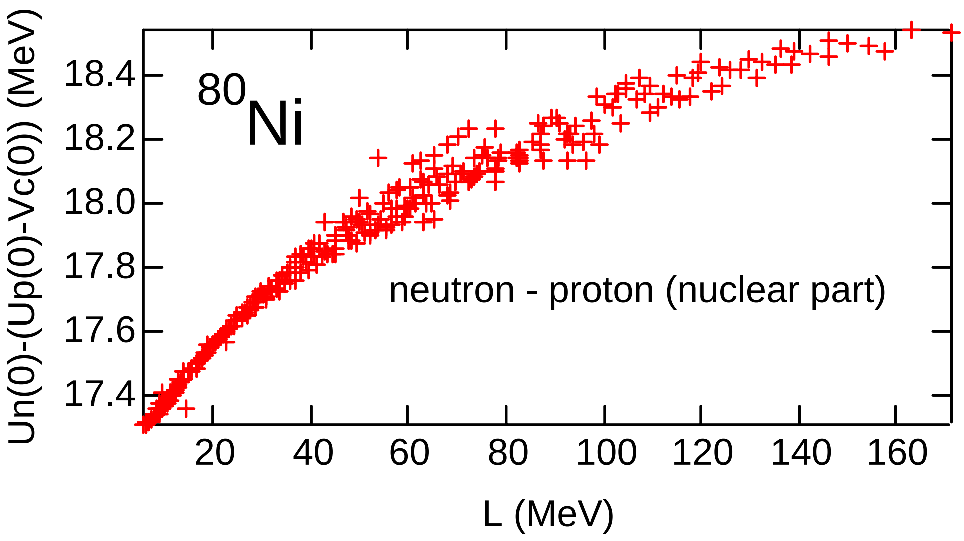


Fig. 5. The potential of the single particle potential of ^{80}Ni (upper box) and its difference between neutrons and protons (lower box). For the proton potential, the Coulomb potential is subtracted to examine its nuclear part only.