Saturation of Asymmetric Nuclear Matter

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Abstract

We examine relations among the parameters characterizing the phenomenological equation of state (EOS) of nearly symmetric, uniform nuclear matter near the saturation density by comparing macroscopic calculations of radii and masses of stable nuclei with the experimental data. The EOS parameters of interest here are the symmetry energy S_0, the density symmetry coefficient L, and the incompressibility K_0 of symmetric nuclear matter at the normal nuclear density. In this study, we also examine the incompressibility of asymmetric matter, which was fixed in a certain functional form in our previous study. This parameter could be important in the description of neutron-rich nuclei and neutron-star matter. In the present study, we treat the incompressibility of the asymmetric matter as a free parameter in fitting the masses and radii, obtain essentially the same EOS parameter values as those in the previous study, and confirm the two important features for symmetry energy; a strong correlation between S_0 and L, and the upper bound of L which is an increasing function of K_0. The present results strongly support the the prediction of the previous study that the matter radii of neutron-rich nuclei depend strongly on L while being almost independent of K_0. This is a feature that will help to determine the L value via systematic measurements of nuclear size.

1 Introduction

The equation of state (EOS) of nuclear matter is the key nuclear property that determines macroscopic nuclear properties such as nuclear masses and radii. The saturation density and energy of symmetric nuclear matter, which consists of equal numbers of neutrons and protons, are determined rather precisely from masses and radii of stable nuclei, in which numbers of neutrons and protons are not very different. In near future, a radioactive ion beam will enable us to measure nuclear masses and radii of heavy nuclei with large neutron excess. In order to make full use of the future experiment for the empirical determination of asymmetric matter EOS, it is important to clarify what kind of EOS properties can be determined from stable nuclei, and what kind of EOS properties can not be determined from stable nuclei but from neutron rich nuclei. In this paper, we focus on the empirical saturation properties of the EOS to be obtained from stable nuclei.

The energy per nucleon near the saturation point of symmetric nuclear matter is generally expressed as [1]

$$w = w_0 + \frac{K_0}{18n_0^2}(n-n_0)^2 + \left[S_0 + \frac{L}{3n_0}(n-n_0) + \frac{K_{asym}}{18n_0^2}(n-n_0)^2\right]\alpha^2. \tag{1}$$

Here w_0, n_0 and K_0 are the saturation energy, the saturation density and the incompressibility of symmetric nuclear matter. The neutron excess is defined as $\alpha = 1 - 2x$ using proton fraction x. The parameters S_0 (the symmetry energy), L (the density symmetry coefficient) and K_{asym} characterize the density dependent symmetry energy $S(n)$ at $n \approx n_0$;

$$S_0 = S(n_0), \tag{2}$$

$$L = 3n_0(dS/dn)_{n=n_0}, \tag{3}$$

$$K_{asym} = 9n_0^2(d^2S/dn^2)_{n=n_0}. \tag{4}$$

From Eq. (1), the saturation density n_s and energy w_s of asymmetric nuclear matter with fixed proton fraction are given, up to the second order of α, by

$$n_s = n_0 - \frac{3n_0L}{K_0}\alpha^2, \tag{5}$$

$$w_s = w_0 + S_0\alpha^2. \tag{6}$$

One useful empirical parameter to characterize the saturation of asymmetric nuclear matter is the slope, y, of the saturation line near $\alpha = 0$ ($x = 1/2$) [1]. It is expressed as

$$y = -\frac{K_0S_0}{3n_0L}. \tag{7}$$

In this paper, we systematically examine empirical relations among the six EOS parameters in Eq. (1). Specifically, we use a parametrized EOS, which is simple but flexible enough to fit non-relativistic and relativistic phenomenological EOS's at $n < 2n_0$. The parameter values are chosen to fit masses and radii of stable nuclei in the Thomas-Fermi approximation.

2 Macroscopic description of nuclei

In constructing a macroscopic nuclear model, we begin with a simple expression for the bulk energy per nucleon [5],

$$w = \frac{3\hbar^2(3\pi^2)^{2/3}}{10m_nn}(n_n^{5/3} + n_p^{5/3}) + (1-\alpha^2)v_s(n)/n + \alpha^2v_n(n)/n, \tag{8}$$

where

$$v_s = a_1n^2 + \frac{a_2n^3}{1+a_3n} \tag{9}$$

and

$$v_n = b_1n^2 + \frac{b_2n^3}{1+b_3n} \tag{10}$$

are the potential energy densities for symmetric nuclear matter and pure neutron matter, and m_n is the neutron mass. Here, replacement of the proton mass m_p by m_n in the proton kinetic energy makes only a negligible difference. Equation (8) can well reproduce the microscopic calculations of symmetric nuclear matter and pure neutron matter by Friedman and Pandharipande [3] and of asymmetric nuclear matter by Lagaris and Pandharipande [4]. Furthermore the expression can also reproduce phenomenological Skyrme Hartree-Fock and relativistic mean field EOS's.

We determine the parameters included in Eqs. (9) and (10) in such a way that they reproduce data on radii and masses of *stable* nuclei. In the limit of $n \to n_0$ and $\alpha \to 0$ ($x \to 1/2$), expression (8) reduces to the usual form (1) [2].

We describe a spherical nucleus of proton number Z and mass number A within the framework of a simplified version of the extended Thomas-Fermi theory [5]. We first write the total energy of a nucleus as a function of the density distributions $n_n(\mathbf{r})$ and $n_p(\mathbf{r})$ according to

$$E = \int d^3r n(\mathbf{r}) w\left(n_n(\mathbf{r}), n_p(\mathbf{r})\right) + F_0 \int d^3r |\nabla n(\mathbf{r})|^2 + \frac{e^2}{2} \int d^3r \int d^3r' \frac{n_p(\mathbf{r}) n_p(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + N m_n + Z m_p, \tag{11}$$

where the first, second and third terms on the right hand side are the bulk energy, the gradient energy with an adjustable constant F_0, and the Coulomb energy, respectively. The symbol $N = A - Z$ denotes the neutron number. Here we ignore shell and pairing effects. We also neglect the contribution to the gradient energy from $|\nabla(n_n(\mathbf{r}) - n_p(\mathbf{r}))|^2$; this contribution makes only a little difference even in the description of extremely neutron-rich nuclei, as clarified in the context of neutron star matter [5].

For the present purpose of examining the macroscopic properties of nuclei such as masses and radii, it is sufficient to characterize the neutron and proton distributions for each nucleus by the central densities, radii and surface diffuseness different between neutrons and protons, as in Ref. [5]. We thus assume the nucleon distributions $n_i(r)$ $(i = n, p)$, where r is the distance from the center of the nucleus, as

$$n_i(r) = \begin{cases} n_i^{\rm in} \left[1 - \left(\dfrac{r}{R_i} \right)^{t_i} \right]^3 , & r < R_i, \\ 0 , & r \geq R_i. \end{cases} \tag{12}$$

Here R_i roughly represents the nucleon radius, t_i the relative surface diffuseness, and $n_i^{\rm in}$ the central number density. The proton distribution of the form (12) can fairly well reproduce the experimental data for stable nuclei such as ^{90}Zr and ^{208}Pb [5].

3 Optimization using smoothed nuclear data

The EOS parameters a_1–b_3 and F_0 are determined from masses and radii of stable nuclei in the same way as in Refs. [2, 5] using the empirical values for nine nuclei on the smoothed β-stability line ranging $25 \leq A \leq 245$ (see Table A.1 in Ref. [5], which is based on Refs. [6, 7]). In contrast to Refs. [2, 5] in which the b_3 value was fixed, the b_3 value is also varied in the present study to have a feeling about K_{asym} dependence. For fixed slope y and incompressibility K_0, such a comparison is made by a usual least squares fitting, which gives rise to an optimal set of the parameters a_1–b_3 and F_0. Here, we set y and K_0 as -1800 MeV fm^3 $\leq y \leq -200$ MeV fm^3 and 180 MeV $\leq K_0 \leq$ 360 MeV; the numerical results for n_0, w_0, S_0, L and F_0 are obtained for about 200 combinations of y and K_0. All of them reproduce the input nuclear data almost equally.

The b_3 value, which is optimized in the present study, is found rather close to the value in the previous study ($b_3 = 1.58632$ fm^3) [2] as shown in Fig. 1. The optimum relations among the EOS parameters obtained in the present study is quite similar to those in the previous study because the EOS parameters at $n = n_0$ are not sensitive to b_3. The parameter b_3 only softens the asymmetric matter EOS at high densities.

As shown in Fig. 2 the present study confirms the empirical correlation between S_0 and L obtained in the previous study [2],

$$S_0 \approx B + CL, \tag{13}$$

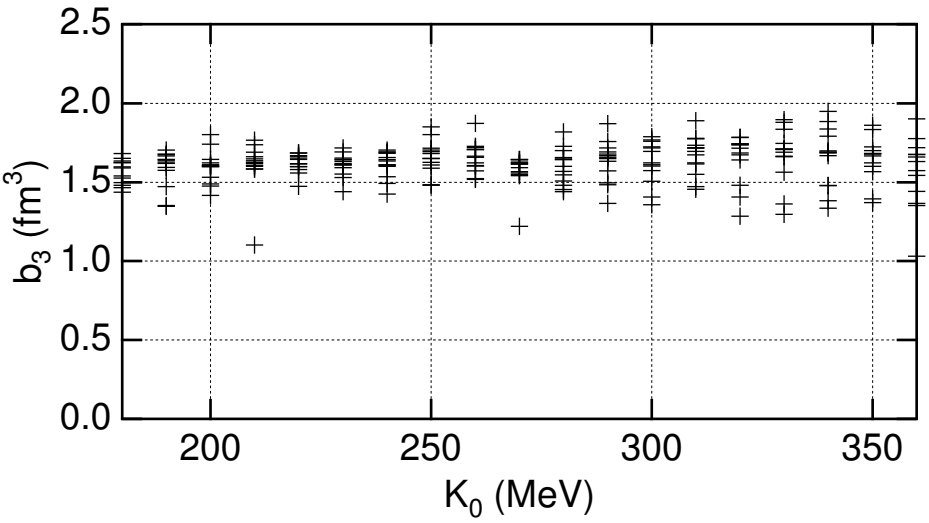

Figure 1: The b_3 value in Eq. (10). This value was fixed in the previous study [2].

with the same values of the coefficients ($B \approx 28$ MeV and $C \approx 0.075$). A similar result, $B = 29$ MeV and $C = 0.1$, was obtained from various Hartree-Fock models with finite-range forces by Farine et al. [8]. As for the saturation of asymmetric nuclear matter, the above correlation is the only information that can be obtained from stable nuclei. This is the reason why there is significant difference among empirical asymmetric matter EOS's although they reproduce properties of stable nuclei.

The saturation energy of symmetric nuclear matter, w_0, always takes a value of -16.1 ± 0.2 MeV. As shown in Fig. 3, the present study obtains the same weak correlation between n_0 and K_0 as the previous study [2]. This is a feature found among non-relativistic phenomenological Skyrme Hartree-Fock EOS's (see Fig. 4 of Ref. [9]).

In Fig. 4, the uncertainties in L and K_0 is represented as a band, which reflects the constraint on (y, K_0). In this band, L increases with increasing y for fixed K_0. The upper bound ($y = -200$ MeV fm^3) reaches a large value of L, which increases with increase in K_0.

4 Summary

About 200 sets of the EOS parameters are systematically obtained from fitting to masses and radii of stable nuclei using a simplified Thomas-Fermi model paying attention to large uncertainties in K_0 and L values.

As for symmetric nuclear matter, the saturation density density n_0 has a weak K_0 dependence while the saturation energy is w_0 is essentially constant.

There is a strong correlation between S_0 and L : $S_0 \approx 28 + 0.075L$ (MeV). However, the L value can not be singled out from stable nuclei although the upper bound of L can be estimated as an increasing function of K_0 from the empirical constraint on the slope of the saturation line. This is all we obtain from stable nuclei about asymmetric nuclear matter. As a consequence, empirical EOS's for asymmetric matter can vary significantly in spite that they reproduce properties of stable nuclei almost equally.

From the present study allowing a wider EOS parameter space, it is found that an artificial

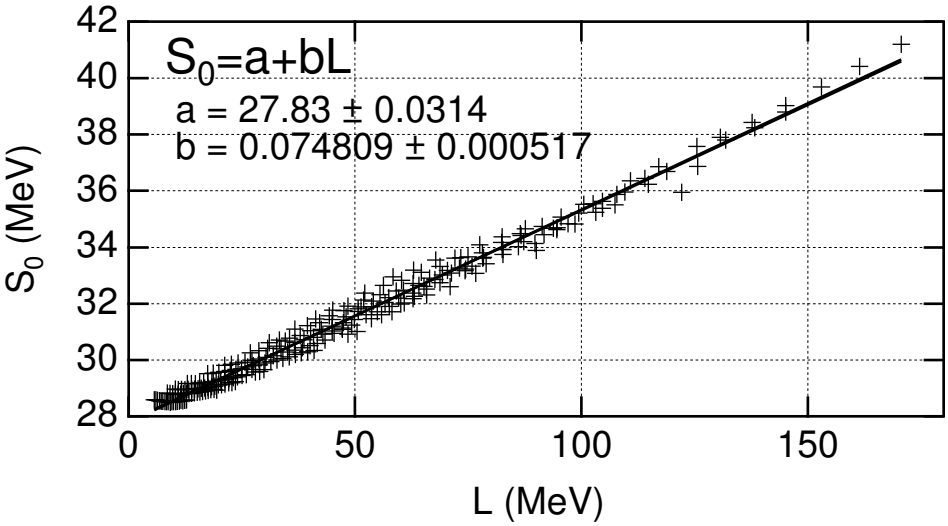

Figure 2: The empirical correlation between S_0 and L obtained in the present study.

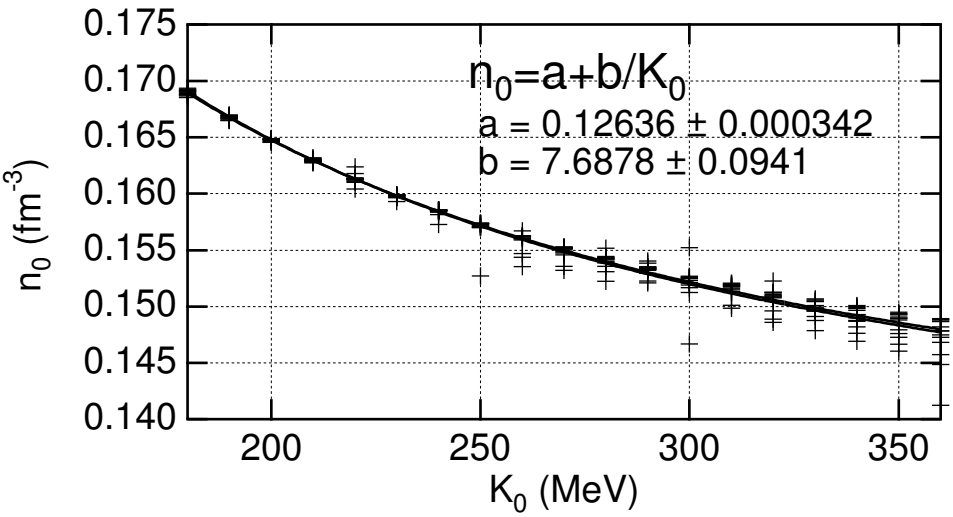

Figure 3: The empirical correlation between n_0 and K_0 obtained in the present study.

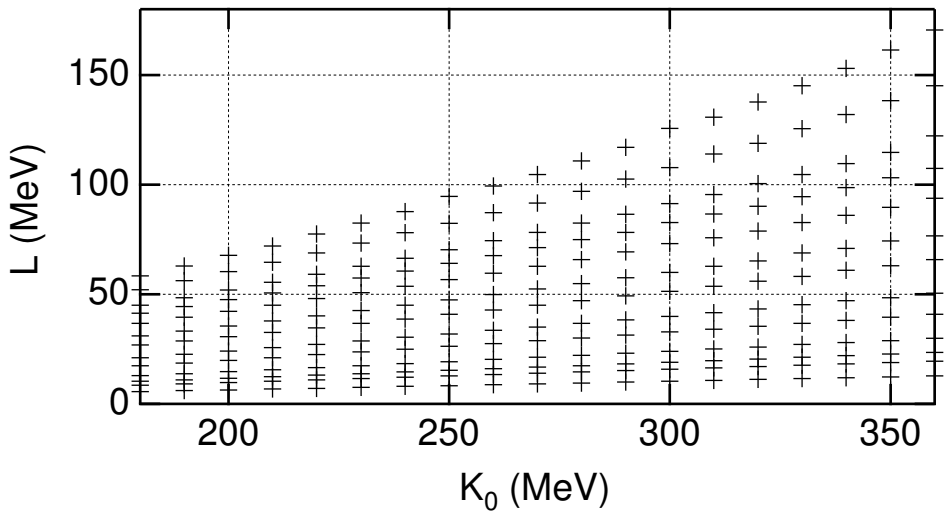

Figure 4: The optimum (L, K_0) values.

constraint in the previous study ($b_3 = 1.586$ fm^3) makes little difference in determining the EOS. Hence, the present results supports the following scenario of our previous study; the L value can not be determined from stable nuclei, but could be determined from neutron-rich nuclei. From these results, we conclude that future systematic measurements of the matter radii of neutron-rich nuclei could help deduce the L value, which in turn could give useful information about the presence of pasta nuclei in neutron star crusts.

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