

# Analysis of Continuum Spectra of $(n, d)$ reactions with Direct Reaction Model

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## Abstract

Higher energy deuteron emission spectra in the continuum region in nucleon induced reactions, i.e.  $(p, d)$  and  $(n, d)$  reactions, are not reproduced well by the usual pre-equilibrium reaction model. It has been suggested that the one-step direct pick-up reaction model gives better predictions for the  $(p, d)$  reactions at incident energies of several tens MeV region. The present study aims to establish a method to analyze the continuum spectra of both the  $(p, d)$  and  $(n, d)$  reactions in the direct reaction scheme with global optical potentials. In this paper, we analyze the  $^{58}\text{Ni}(p, d)^{57}\text{Ni}$  and  $^{nat}\text{Fe}(n, d)^{nat}\text{Mn}$  reactions in ensemble by this model.

## I. Introduction

There have been many works on one-nucleon transfer reactions. However, works on the  $(n, d)$  reaction have been yet scarce. As the  $(n, d)$  reaction data are not easy to be measured experimentally, it is desired to prepare a model which gives a reliable theoretical prediction, and to use it as a substitute of experimental results.

Some theoretical models have been proposed to study the continuum spectra excited by the  $(n, d)$  reactions [1–3], which can not reproduce well the experimental data [4].

For the  $(p, d)$  reaction continuum spectra, Syafarudin et al. [5] have given a procedure, which can reproduce well the spectra of the one-nucleon transfer reactions in higher emission energy region. The present work for the  $(n, d)$  reactions is an extension of the  $(p, d)$  reaction analyses. We have applied here the similar procedure to analyze the  $(n, d)$  reaction data with global optical potentials [6].

To predict the direct reaction continuum spectra of the  $(p, d)$  reaction, we adopted an approach suggested by Lewis [7]. According to this model, continuum spectra in the direct reaction scheme are given as a result of nuclear damping. From several studies on the direct reaction scheme, Matoba et al. reached eventually a decision to solve this critical problem [8, 9]. They assumed continuum spectra as an incoherent sum of all shell contribution and adopted an asymmetric Lorentzian form for the response function in the DWBA-based cross sections calculation and the experimental  $(p, d)$  double differential cross sections are predicted. In the present work the same method is applied for analysis of the  $(n, d)$  reactions to reproduce the double-differential cross-section. From this work, we can realize that this method is applicable not only for the  $(p, d)$  reactions, but also for the  $(n, d)$  reactions.

## II. Experimental data

### 1. $(p, d)$ reactions

The experiments were performed at the TIARA facility of JAERI. A proton beam of 68MeV from the AVF cyclotron was lead to the HB-1 beam line. Energy distributions of light ions emitted from the target were measured using a  $\Delta E$ -E counter telescope, which consisted of two thin silicon  $\Delta E$ -detectors and a CsI(Tl) E-detector with photo-diode readout. Details of the experimental procedure and the results have been reported in ref [10].

## 2. $(n, d)$ reactions

The experiments were performed at the neutron source  ${}^7\text{Li}(p, n)$  of the TIARA facility of JAERI. A spectrometer, which consisted of three counter telescopes mounted on a vacuum chamber to reduce the energy loss of secondary particles and charged particles in the air was used. Details of the experimental procedure and the results have been reported in ref [11].

## III. Theoretical Analysis

In the present method, the theoretical calculations of the double differential cross-sections have been done by considering a direct reaction model as an incoherent sum of the direct reaction components, which are based on DWBA predictions and expressed as below:

$$\frac{d^2\sigma}{d\Omega dE} = 2.30 \sum_{l,j} \left[ \frac{C^2 S_{l,j}(E)}{2j+1} \times \left( \frac{d\sigma}{d\Omega} \right)_{l,j}^{DW}(E) \right] \quad (1)$$

where  $d\sigma/d\Omega|_{l,j}^{DW}(E)$  is the cross-section calculated by a DWBA code DWUCK [12] and  $C^2 S_{l,j}(E)$ , the spectroscopic factor expressed as-

$$C^2 S_{l,j}(E) = \left( \sum C^2 S_{l,j} \right) \times f_{l,j}(E) \quad (2)$$

where  $\sum C^2 S_{l,j}$  is the sum of the spectroscopic factors of all the predicted states and the distribution of strength function over the spectra is obtained by using an asymmetric Lorentzian function [8, 9, 13]

$$f_{l,j} = \frac{n_0}{2\pi} \frac{\Gamma(E)}{(|E - E_F| - E_{l,j})^2 + \Gamma^2(E)/4} \quad (3)$$

and

$$\int_0^\alpha f_{l,j}(E) dE = 1 \quad (4)$$

where  $n_0$  is the renormalization constant and  $E_F$  is the Fermi energy. The Fermi energy can be calculated by using an empirical formula given in [14]. The sums of spectroscopic factors and the centroid energies ( $E_{l,j}$ ) for  $J = l \pm \frac{1}{2}$  shell orbits have been estimated by using BCS calculations. In these calculations, single particle energies required to calculate the centroid energy are calculated by the prescription of Bohr and Motelson [15]. Spreading width ( $\Gamma$ ) is expressed by a function proposed by Brown and Rho [16] and by Mahaux and Sartor [13], as,

$$\Gamma(E) = \frac{\epsilon_0 (E - E_F)^2}{(E - E_F)^2 + E_0^2} + \frac{\epsilon_1 (E - E_F)^2}{(E - E_F)^2 + E_1^2} \quad (5)$$

where  $\epsilon_0$ ,  $\epsilon_1$ ,  $E_0$  and  $E_1$  are constants which express the effects of nuclear damping in the nucleus [8]. The estimated parameters [8] are

$$\begin{aligned} \epsilon_0 &= 19.4 \text{ (MeV)}, & E_0 &= 18.4 \text{ (MeV)}, \\ \epsilon_1 &= 1.40 \text{ (MeV)}, & E_1 &= 1.60 \text{ (MeV)}. \end{aligned} \quad (6)$$

The sum rule of the spectroscopic factors of nucleon orbits for  $T \pm \frac{1}{2}$  isospin states is estimated with a simple shell model prescription [17]

$$\sum C^2 S_{l,j} = \begin{cases} n_n(l, j) - \frac{n_p(l, j)}{2T+1} & \text{for } T_< = T - \frac{1}{2} \\ \frac{n_p(l, j)}{2T+1} & \text{for } T_> = T + \frac{1}{2} \end{cases} \quad (7)$$

where  $n_{n(l,j)}$  and  $n_{p(l,j)}$  are the numbers of neutrons and protons respectively for each  $(l, j)$  orbit and  $T$  is the isospin of the target nucleus.

Table 1

Optical model parameters used in the DWBA calculations.

 $^{58}\text{Ni}(p, d)^{57}\text{Ni}$ :

Particle	V (MeV)	r (fm)	a (fm)	$r_c$ (fm)	$W_v$ (MeV)	$W_s$ (MeV)	$r'$ (fm)	$a'$ (fm)	$V_{so}$ (MeV)	$r_{so}$ (fm)	$a_{so}$ (fm)
Proton	32.11	1.20	0.67	1.26	7.36	3.11	1.28	0.54	4.51	1.02	0.59
Deuteron	a	1.20	0.67	1.26	b	c	1.28	0.54	d	1.02	0.59
Neutron	e	1.25	0.65								

 $^{54}\text{Fe}(n, d)^{53}\text{Mn}$ :

Particle	V (MeV)	r (fm)	a (fm)	$r_c$ (fm)	$W_v$ (MeV)	$W_s$ (MeV)	$r'$ (fm)	$a'$ (fm)	$V_{so}$ (MeV)	$r_{so}$ (fm)	$a_{so}$ (fm)
Neutron	30.04	1.20	0.67		7.11	2.38	1.28	0.54	4.33	1.01	0.59
Deuteron	a	1.20	0.67	1.26	b	c	1.28	0.54	d	1.01	0.59
Proton	e	1.25	0.65	1.26							

 $^{56}\text{Fe}(n, d)^{55}\text{Mn}$ :

Particle	V (MeV)	r (fm)	a (fm)	$r_c$ (fm)	$W_v$ (MeV)	$W_s$ (MeV)	$r'$ (fm)	$a'$ (fm)	$V_{so}$ (MeV)	$r_{so}$ (fm)	$a_{so}$ (fm)
Neutron	29.65	1.20	0.67		7.11	2.30	1.28	0.54	4.34	1.02	0.59
Deuteron	a	1.20	0.67	1.26	b	c	1.28	0.54	d	1.02	0.59
Proton	e	1.25	0.65	1.26							

 $^{57}\text{Fe}(n, d)^{56}\text{Mn}$ :

Particle	V (MeV)	r (fm)	a (fm)	$r_c$ (fm)	$W_v$ (MeV)	$W_s$ (MeV)	$r'$ (fm)	$a'$ (fm)	$V_{so}$ (MeV)	$r_{so}$ (fm)	$a_{so}$ (fm)
Neutron	29.46	1.20	0.67		7.11	2.26	1.28	0.54	4.34	1.02	0.59
Deuteron	a	1.20	0.67	1.26	b	c	1.28	0.54	d	1.02	0.59
Proton	e	1.25	0.65	1.26							

 $^{58}\text{Fe}(n, d)^{57}\text{Mn}$ :

Particle	V (MeV)	r (fm)	a (fm)	$r_c$ (fm)	$W_v$ (MeV)	$W_s$ (MeV)	$r'$ (fm)	$a'$ (fm)	$V_{so}$ (MeV)	$r_{so}$ (fm)	$a_{so}$ (fm)
Neutron	29.28	1.20	0.67		7.11	2.22	1.28	0.54	4.34	1.02	0.59
Deuteron	a	1.20	0.67	1.26	b	c	1.28	0.54	d	1.02	0.59
Proton	e	1.25	0.65	1.26							

Nonlocality parameters (fm)			Finite-range parameter (fm)			spin orbit term $\lambda = 25$	
Proton	0.85		0.621				
Neutron	0.85		0.621				
deuteron	0.54						

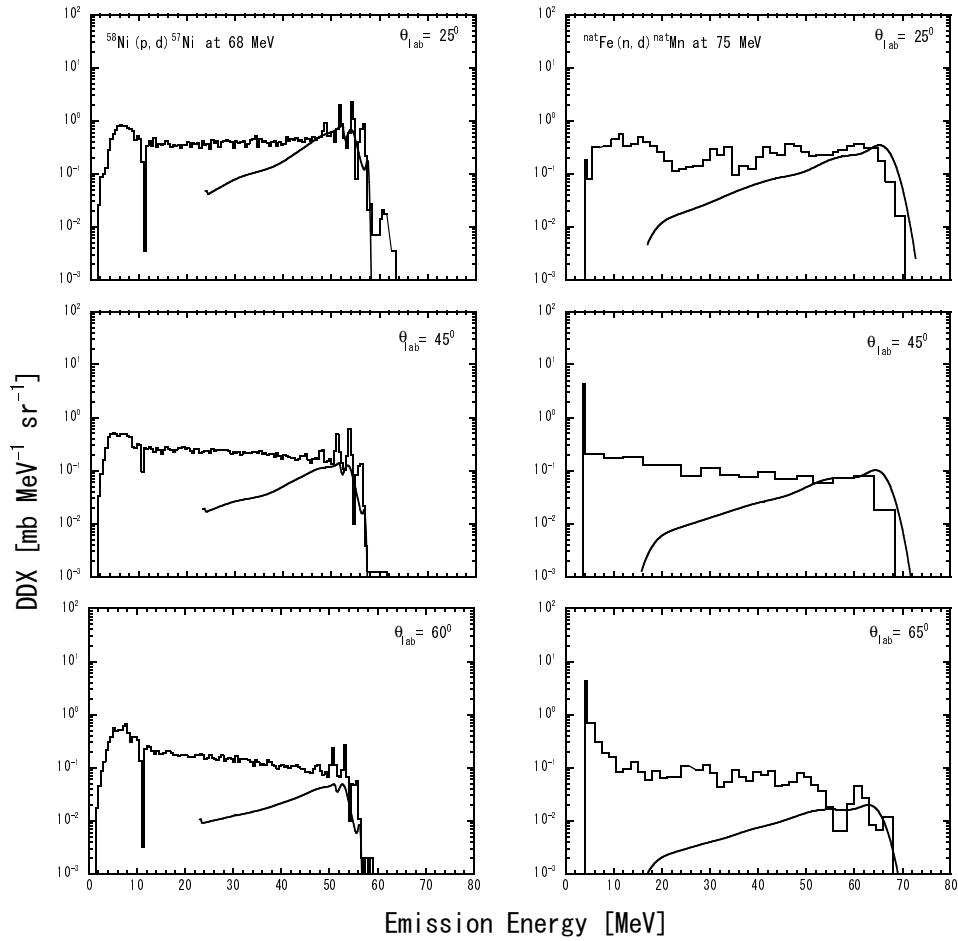
<sup>a</sup>  $V = V(\text{proton}) + V(\text{neutron})$ , See ref. [6] for  $V(\text{proton})$  and  $V(\text{neutron})$ .<sup>b</sup>  $W_v = W_v(\text{proton}) + W_v(\text{neutron})$ , See ref. [6] for  $W_v(\text{proton})$  and  $W_v(\text{neutron})$ .<sup>c</sup>  $W_s = W_s(\text{proton}) + W_s(\text{neutron})$ , See ref. [6] for  $W_s(\text{proton})$  and  $W_s(\text{neutron})$ .<sup>d</sup>  $V_{so} = V_{so}(\text{proton}) + V_{so}(\text{neutron})$ , See ref. [6] for  $V_{so}(\text{proton})$  and  $V_{so}(\text{neutron})$ .<sup>e</sup> Well depth adjusted to fit the separation energy.

This sum rule of each orbit is suitable for  $(p, d)$  reaction but for  $(n, d)$  reaction, we considered no contribution for  $n_{n(l,j)}$  i.e. no contribution for IAS in the spectrum. So we applied 100% contribution for the spectra only for  $n_{p(l,j)}$  and did some modification of the above sum rule equation i.e.

$$\sum C^2 S_{l,j} = \frac{n_{p(l,j)}}{2T + 1} \quad (8)$$

#### IV. Results and Discussion

Experimental and theoretical double differential cross-sections, for the  $^{58}\text{Ni}(p, d)^{57}\text{Ni}$  and the  $^{nat}\text{Fe}(n, d)^{nat}\text{Mn}$  reactions at 68 MeV and 75 MeV, respectively are shown in fig 1. For natural Fe target, we consider isotopes ( $^{54}\text{Fe}$ ,  $^{56}\text{Fe}$ ,  $^{57}\text{Fe}$  and  $^{58}\text{Fe}$ ) with natural abundances and calculate their contributions to double-differential cross-sections. Table 1 shows the optical model parameters used in the DWBA calculations for the  $^{58}\text{Ni}(p, d)^{57}\text{Ni}$  and  $^{nat}\text{Fe}(n, d)^{nat}\text{Mn}$  reactions. In fig 1, thin solid curves represent the experimental spectra and thick one the theoretical. The calculated spectra of both the  $(p, d)$  and  $(n, d)$  reactions obtained from the same method of calculation are in good agreement with the experimental ones in the higher energy region. To compensate the experimental energy resolutions for the  $(p, d)$  and  $(n, d)$  reactions, a convolution integration was applied to the theoretical cross-section with experimental resolution.



**Fig. 1**  $^{58}\text{Ni}(p, d)$  DDX data(left) and  $^{nat}\text{Fe}(n, d)$  DDX data(right) at 68 MeV and 75 MeV respectively. The thin solid curves show the result of experimental data and the thick one the theoretical.

#### V. Conclusion

The  $^{58}\text{Ni}(p, d)^{57}\text{Ni}$  and  $^{nat}\text{Fe}(n, d)^{nat}\text{Mn}$  reactions data have been analyzed here with the same method of calculations.

The theoretical calculations can reproduce well experimental spectra of forward angles ( $25^0, 45^0$ ), at high outgoing energies. But for the spectra at backward angles ( $60^0$  for  $^{58}\text{Ni}$  and  $65^0$  for  $^{nat}\text{Fe}$ ), the calculated results are somewhat underestimated. It is thus possible that for the backward angles there may be some contribution from the pre-equilibrium reaction process.

As a whole, a fairly good overall agreement is found between the theoretical and experimental spectra in both the magnitude and shape of double-differential cross-section. So from all the above consideration we can conclude that this theoretical method is suitable not only for the  $(p, d)$  but also for  $(n, d)$  reactions. Further studies of the  $(p, d)$  and  $(n, d)$  reactions for the targets with atomic mass ranging 27-209 are now planned to make this theoretical procedure more global.

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