

# Percolation-fission model study of the fragment mass distribution for the 1 GeV proton induced reaction

Masahiko KATSUMA<sup>1</sup>, Hiroshi KOBAYASHI<sup>1</sup>, Toshinobu SASA<sup>2</sup>  
and Tetsuo SAWADA<sup>1</sup>

<sup>1</sup> *Research Laboratory for Nuclear Reactors, Tokyo Institute of Technology,  
Meguro-ku, Tokyo 152-8550, Japan*

<sup>2</sup> *Tokai Research Establishment, Japan Atomic Energy Research Institute,  
Tokai-mura Naka-gun, Ibaraki 319-1195, Japan*

The 1 GeV proton induced reaction on <sup>208</sup>Pb targets is analyzed by using the percolation model combined with the Atchison fission model. The fragment mass distribution and the isotopic production cross sections obtained from our model are compared with the experimental data. The trends of the fragment mass distribution for the 1 GeV proton induced reaction can be reproduced by our calculation in some degree. The order of magnitude for the calculated isotopic production cross sections at the calculated peak positions is similar to that of the experimental peak values. The calculated peak positions of the isotopic production cross sections are shifted to the heavier region than those of the experimental data.

## 1. Introduction

Recently, in the nuclear engineering field, research activities have been launched for evaluation of yields of spallation products in target systems of accelerator driven systems (ADS) for transmutation of radioactive wastes. From the viewpoint of radiation safety, it has been necessary to estimate the amount of radiotoxicity due to rare-earth elements produced in various targets.<sup>1)</sup> However, we are in the lack of the experimental data for fragment mass distributions in proton induced reactions at incident energies around 1 GeV. In this situation, theoretical studies of spallation and fission in the proton induced reactions give us some helpful information about estimating of reaction products.

Conventionally, the theoretical studies of spallation and fission in the proton induced reactions have been performed by combining intranuclear cascade, evaporation and fission models.<sup>2,3)</sup> These studies have not perfectly accomplished yet, especially, for the reproductions of fragment mass distributions at incident energies around 1 GeV. By contrast, the percolation model works well to describe mass and energy distributions of fragments produced from multifragmentation process in pA ( $> 10$  GeV) reactions.<sup>4-8)</sup> In order to study reaction products in proton induced reactions below 10 GeV, no modification to

the percolation model has been made so as to consider the possibility of spallation-fission competition in fissionable heavy nuclei.

In this paper, we apply the percolation model<sup>4-7)</sup> to the analyses of the 1 GeV proton induced reaction on  $^{208}\text{Pb}$  targets,<sup>9,10)</sup> and we make some modification to the percolation model. At the incident energy  $E_{\text{inc}} = 1$  GeV, we assume that fission process competes with percolation-like decay process at the stage of nuclear de-excitations. As for the fission process, we adopt the Atchison fission model<sup>11,12)</sup> that is frequently implemented in the combined model of intranuclear cascade, evaporation and fission. The percolation model combined with the Atchison fission model will be called the percolation-fission (PF) model below. The GSI experimental data<sup>9)</sup> of the fragment mass distribution and the isotopic production cross sections for spallation and fission products in the 1 GeV proton induced reaction will be compared with the results of the PF model.

The PF model is described in Sec. 2. The comparison between the results of the PF model calculations and the experimental data is shown in Sec. 3. The summary is in Sec. 4.

## 2. Percolation-fission model

In the PF model, target nuclei are constructed by an approximately spherical representation on a simple-cubic lattice or a hexagonal lattice.<sup>6,7)</sup> The nucleons are taken to occupy the sites of a simple-cubic lattice or a hexagonal lattice. The site distances between two nucleons are adjusted to reproduce the root mean squared radius of target nuclei. Neighboring pairs of the sites are connected by bonds. We assume that bonds between two protons and bonds between two neutrons are initially broken in the case of a hexagonal lattice for target nuclei.

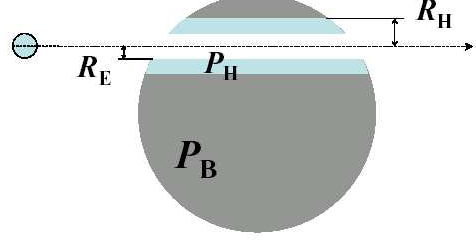
For a given impact parameter  $b$ , the geometry of pre-decay nuclei is schematically illustrated in Fig. 1. To construct the pre-decay nuclei, we remove nucleons on a cylindrical channel with a radius  $R_E$  from the lattice. We assume that bonds between two nucleons on a cylindrical region with a radius  $R_H$  are easily broken, because nucleons on it acquire higher energies from the incident particle shooting. The cylindrical region with the radius  $R_H$  is called the hot region.

The bond breaking probability  $P_B(b)$  of the lattice, except for the hot region, is defined as follows,

$$P_B(b) = \frac{P_0 \int_{-\infty}^{+\infty} \rho[\mathbf{R}(b)] d\mathbf{R}}{\int_{-\infty}^{+\infty} \rho[\mathbf{R}(0)] d\mathbf{R}}. \quad (1)$$

The  $P_0$  is the bond breaking probability of the central collision,  $b = 0$ . The  $\rho(r)$  is the matter density distribution of target nuclei, which is given in a conventional Woods-Saxon form with  $\rho_0 = 0.159 \text{ fm}^{-3}$ ,  $r_0 = 1.12 \text{ fm}$ , and  $a_0 = 0.55 \text{ fm}$  for  $^{208}\text{Pb}$ . The bond breaking probability  $P_B(b)$  decreases as  $b$  increases. The incident proton with an impact parameter

$b$  is randomly generated with Monte Carlo algorithm. The bond breaking probability  $P_H$  in the hot region of nuclei is independent of impact parameters. In our model,  $R_E$ ,  $R_H$ ,  $P_0$  and  $P_H$  are the adjustable parameters in dependence on incident energies to fit the experimental data.



**Fig. 1** geometry of pre-decay nuclei.

In the PF model, excited nuclei after having been bombarded by an incident proton go on in the percolation-like decay process, or the fission decay process. A random number  $x$  is generated, and then, the decay process is selected from the following equations,

$$\begin{aligned} x &> P_{\text{fis}}(E_{\text{ex}}, A_i, Z_i) && \text{(Percolation-like decay),} \\ x &\leq P_{\text{fis}}(E_{\text{ex}}, A_i, Z_i) && \text{(Fission decay).} \end{aligned} \quad (2)$$

Here,  $P_{\text{fis}}(E_{\text{ex}}, A_i, Z_i)$  is the fission probability, in dependent on an excitation energy  $E_{\text{ex}}$ , a mass number  $A_i$ , and a proton number  $Z_i$  of the pre-decay nuclei. The fission probability is given by the Atchison fission model, and it is expressed as follows,

$$P_{\text{fis}}(E_{\text{ex}}, A_i, Z_i) = \frac{C}{1 + \Gamma_n/\Gamma_{\text{fis}}}, \quad (3)$$

where  $\Gamma_n$  and  $\Gamma_{\text{fis}}$  are the widths for neutron emission and fission for the pre-decay nuclei. In the PF model, the  $C$  is the reduction factor of the fission probability which is in inverse proportion to the number of emitted particles in percolation-like decay process,  $C = (1 - E'_{\text{cm}}/E_{\text{ex}})/N_e$ . The  $E'_{\text{cm}}$  is the center-of-mass energy after decay.

When the percolation-like decay process is selected from Eq. (2), a random number  $x$  is generated with Monte-Carlo algorithm. And then, whether each bond of the lattice is broken or not is decided from the following relations,

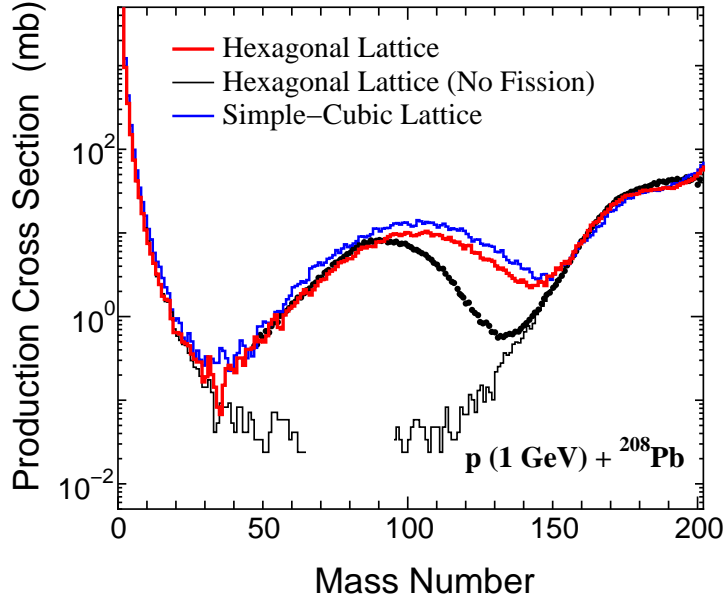
$$\begin{aligned} x &\leq P_B(b) && \text{(Broken),} \\ x &> P_B(b) && \text{(Unbroken).} \end{aligned} \quad (4)$$

It should be noted that  $P_B(b)$  is the same for each bond except for the hot region, independent of its position on the lattice. The relations in the hot region are similar to those for  $P_B(b)$ ,

$$\begin{aligned} x &\leq P_H && \text{(Broken),} \\ x &> P_H && \text{(Unbroken).} \end{aligned} \quad (5)$$

Note that  $P_H$  is the same for each bond in the hot region, independent of its position on the lattice and of the impact parameter. In the end of calculations for the percolation-like decay, we count the number of isolated fragments which nucleons are still connected by bonds.

When the fission decay process is selected from Eq. (2), mass and charge distributions for two fission nuclei are given from the empirical formulae in the Atchison fission model.



**Fig. 2** Fragment mass distribution for the 1 GeV proton induced reaction on  $^{208}\text{Pb}$  targets. The red and blue histograms are the results obtained from the PF model with the hexagonal lattice and the simple-cubic lattice, respectively. The black histogram is the result of the simple percolation model without the fission process. The solid circles are the experimental production cross sections, which are taken from Fig.18 of Ref. 9.

### 3. Results

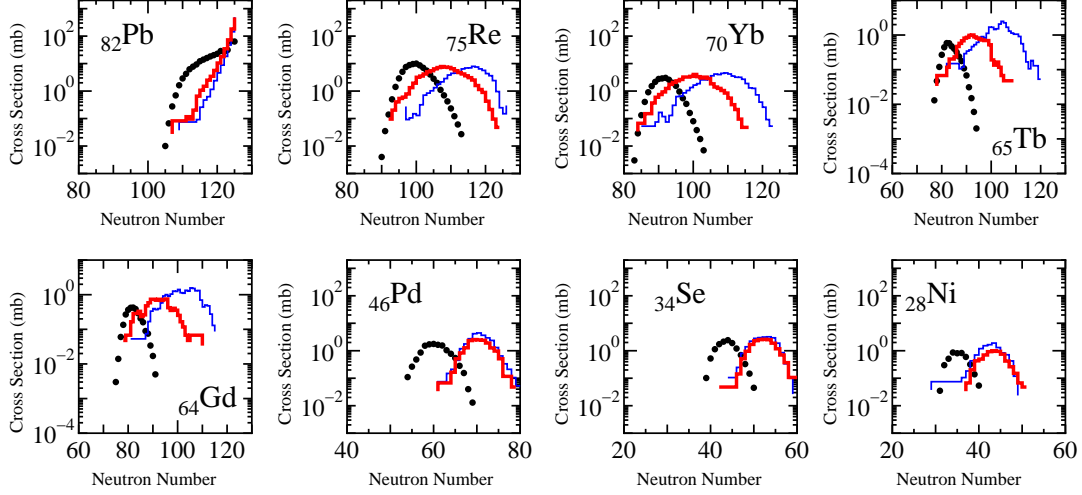
Figure 2 shows the results of the fragment mass distribution for the 1 GeV proton induced reaction on  $^{208}\text{Pb}$  targets. The red and blue histograms are the results obtained from the PF model with the hexagonal lattice and the simple-cubic lattice, respectively. The parameter values of the PF model are listed in Table 1. In Fig. 2, one can see the small difference between the red and blue histograms. The characteristic trends in the experimental production cross section are reproduced by the red and blue histograms. However, in the mass number  $90 < A < 150$  region, the production cross sections of the red and blue histograms are larger compared with that of the experimental data.

The black histogram in Fig. 2 is the result obtained from the simple percolation model with the hexagonal lattice by using the same parameters as those of the red histogram. Namely, the black histogram is the result without the Atchison fission model. In the mass number  $< 30$  and  $> 150$  regions, the black histogram is almost similar to the red histogram. By contrast, the production cross section in the mass number  $= 40$  to  $150$  region is dominated by the fission process. This figure means that the fission process should be combined with the percolation model to study the 1 GeV proton induced reaction.

Figure 3 shows the isotopic production cross sections for the 1 GeV proton induced reaction on  $^{208}\text{Pb}$  targets as a function of neutron numbers. The calculated isotopic pro-

**Table 1** The parameters of the PF model with the simple-cubic lattice and the hexagonal lattice.

Lattice	$P_0$	$P_H$	$R_E$	$R_H$
Simple-cube	0.5	0.6	1.0	4.2
Hexagon	0.5	0.6	1.0	3.5



**Fig. 3** Isotopic production cross sections for the 1 GeV proton induced reaction on  $^{208}\text{Pb}$  targets as a function of neutron numbers. The red and blue histograms are the results obtained from the PF model with the hexagonal lattice and the simple-cubic lattice, respectively. The solid circles are the experimental isotopic production cross sections.<sup>9)</sup>

duction cross sections are compared with the experimental data.<sup>9)</sup> We select eight elements of the experimental data,  $^{82}\text{Pb}$ ,  $^{75}\text{Re}$ ,  $^{70}\text{Yb}$ ,  $^{65}\text{Tb}$ ,  $^{64}\text{Gd}$ ,  $^{46}\text{Pd}$ ,  $^{34}\text{Se}$ , and  $^{28}\text{Ni}$ , to see the features of the PF model. The red and blue histograms are the results obtained from the PF model with the hexagonal lattice and the simple-cubic lattice, respectively. In Fig. 3, the order of magnitude for the calculated isotopic production cross sections at the calculated peak positions is similar to that of the experimental peak values. The calculated peak positions of the isotopic production cross sections are shifted to the heavier region than those of the experimental data. For the five heavy elements in Fig. 3, the peak positions obtained from the PF model with the hexagonal lattice are better than those obtained from the simple-cubic lattice. The deviation between the hexagonal lattice and simple-cubic lattice would originate from the difference of the initial bonds. For the three light elements, the red histograms are almost similar to the blue ones.

#### 4. Summary

We have applied the percolation model to the analyses of the 1 GeV proton induced reaction on  $^{208}\text{Pb}$  targets. In order to investigate this reaction, we have made some modification to the percolation model, which is combined with the Atchison fission model. The simple-cubic lattice and the hexagonal lattice are adopted as the nuclear lattice of  $^{208}\text{Pb}$  nuclei. We have compared our results obtained from the PF model with the GSI experimental data.

As shown in Fig. 2, the PF model can reproduce the trends of the fragment mass distribution for the 1 GeV proton induced reaction in some degree. In Fig. 3, the order of

magnitude for the calculated isotopic production cross sections is similar to that of the experimental data. The calculated peak positions of the isotopic production cross sections are shifted to the heavier region than those of the experimental data. As for the five heavy elements in Fig. 3, the peak positions obtained from the PF model with the hexagonal lattice are better than those obtained from the simple-cubic lattice.

The PF model should be improved further in order to make better reproductions in the near future. However, the results in this paper mean that the PF model would become a useful tool to analyze production cross sections in spallation and fission.

## Acknowledgments

We would like to express our sincere thanks to Professor A. Ohnishi and Mr. S. Yamaguchi for useful comments and giving us the numerical calculation program of the non-equilibrium percolation model. One of the authors (M.K.) thanks Professor N. Yamano for encouragement and comments. This job was funded by Ministry of Education, Culture, Sports, Science and Technology.

## References

- [1] M. Saito, A. Stankovskii, V. Artisyuk, *et al.*, “Radiological hazard of spallation products in accelerator-driven system,” *Nucl. Sci. Eng.*, **142**, 22 (2002).
- [2] M. Lindner and A. Turkevich, “Competition between fission and neutron emission in excited heavy nuclei,” *Phys. Rev.*, **119**, 1632 (1960).
- [3] H.W. Bertini, “Low-energy intranuclear cascade calculation,” *Phys. Rev.*, **131**, 1801 (1963).
- [4] W. Bauer, “Extraction of signals of a phase transition from nuclear multifragmentation,” *Phys. Rev.*, **C38**, 1297 (1988).
- [5] W. Bauer, U. Post, D.R. Dean and U. Mosel, “The nuclear lattice model of proton-induced multifragmentation reactions,” *Nucl. Phys.*, **A452**, 699 (1986).
- [6] W. Bauer, D.R. Dean, U. Mosel and U. Post, “New approach to fragmentation reactions: The nuclear lattice model,” *Phys. Lett.*, **B150**, 53 (1985).
- [7] S. Yamaguchi, Master thesis, Hokkaido University (2004); S. Yamaguchi and A. Ohnishi, “Radial expansion effects on spallation IMF formation,” *Prog. Theor. Phys. Suppl.*, **156**, in press (2004).
- [8] Y. Hirata, A. Ohnishi, Y. Nara, *et al.*, “Sideward peak of intermediate mass fragments in high energy proton induced reactions,” *Nucl. Phys.*, **A707**, 193 (2002).
- [9] T. Enqvist, W. Wlazole, P. Armbruster, *et al.*, “Isotopic yields and kinetic energies of primary residues in 1 A GeV  $^{208}\text{Pb} + \text{p}$  reactions,” *Nucl. Phys.*, **A686**, 481 (2001).
- [10] W. Wlazole, T. Enqvist, P. Armbruster, *et al.*, “Cross sections of spallation residues produced in 1A GeV  $^{208}\text{Pb}$  on proton reactions,” *Phys. Rev. Lett.*, **84**, 5736 (2000).
- [11] F. Atchison, “Spallation and fission in heavy metal nuclei under medium energy proton bombardment,” *Jül-Conf-34 KFA-Jülich*, Germany, p.17 (1980); F. Atchison, “A treatment of fission for HETC,” *Proceedings of a Specialists Meeting Issy-Les-Moulineaux*, France, p.199 (1994).
- [12] S. Furihata, “Development of a generalized evaporation model and study of residual nuclei productions,” Ph.D. thesis, Tohoku University (2003).