Density distributions and form factors in neutron-rich nuclei

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Results of charge form factors calculations for several unstable neutron-rich isotopes of light, medium and heavy nuclei (He, Li, Ni, Kr, Sn) are presented and compared to those of stable isotopes in the same isotopic chain. For the lighter isotopes (He and Li) the proton and neutron densities are obtained within a microscopic large-scale shell-model (LSSM), while for heavier ones Ni, Kr and Sn the densities are calculated in deformed self-consistent mean-field Skyrme HF+BCS method. We also compare proton densities to matter densities together with their rms radii and diffuseness parameter values. Whenever possible comparison of form factors, densities and rms radii with available experimental data is also performed. Calculations of form factors are carried out both in plane wave Born approximation (PWBA) and in distorted wave Born approximation (DWBA). These form factors are suggested as predictions for the future experiments on the electron-radioactive beam colliders where the effect of the neutron halo or skin on the proton distributions in exotic nuclei is planned to be studied and thereby the various theoretical models of exotic nuclei will be tested.

1 Introduction

The scattering of particles and ions from nuclei has provided along the years invaluable information on charge, matter, current and momentum distributions of stable isotopes. At present, efforts are devoted to investigate with such probes highly unstable isotopes at radioactive nuclear beam facilities. Concerning the charge distributions of nuclei, it is known that their most accurate determination can be obtained from electron-nucleus scattering. For the case of exotic nuclei the corresponding charge densities are planned to be obtained by colliding electrons with these nuclei in storage rings. As shown in the NuPECC Report [1], a first technical proposal for a low-energy electron-heavy-ion collider made at JINR (Dubna) has been further developed and incorporated in the GSI physics program [2] along with the plan for the electron-ion collider at the MUSES facility at RIKEN [3]. Several interesting and challenging issues can be analyzed by the mentioned electron scattering experiments. One of them is to study how the charge distribution evolves with increasing neutron number (or isospin) at fixed proton number. The question remains up to what extent the neutron halo or skin may trigger sizable changes of the charge root-mean-square (rms) radius, as well as of the diffuseness in the peripherical region of the charge distribution. This point may then be very important for understanding the neutron-proton interaction in the nuclear medium. To this end the preliminary theoretical calculations of the charge form factors of neutron-rich exotic nuclei can serve as a challenge for future experimental works and thus, for accurate determination of the charge distributions in

these nuclei. This can be a test of the different theoretical models used for predicting charge distributions.

In recent years theoretical work has been done along these lines focusing on halo nuclei. Various existing theoretical predictions for the charge distributions in light exotic nuclei ^{6,8}He, ^{11}Li, ^{14}Be, ^{17,19}B have been used for calculations of charge form factors [4] within the PWBA.

In our recent work [5] in comparison with [4] we extend the range of exotic nuclei for which charge form factors are calculated. Along with the new calculations for He and Li isotopes, we present results on charge form factors of several unstable isotopes of medium (Ni) and heavy (Kr and Sn) nuclei and compare them to those of stable isotopes in the same isotopic chain. The isotopes of Ni and Sn are chosen because they have been indicated in Ref. [3] as first candidates accessible for the charge densities and rms radii determination and as key isotopes for structure studies of unstable nuclei at the electron-radioactive-ion collider in RIKEN. We calculate the charge form factors not only within the PWBA but also in DWBA by the numerical solution of the Dirac equation [6, 7, 8] for electron scattering in the Coulomb potential of the charge distribution of a given nucleus. Also, now we do not neglect neutrons, as was done in Ref. [4].

2 The Theoretical Scheme

The nuclear charge form factor $F_{ch}(q)$ has been calculated as follows

$$F_{ch}(q) = \left[F_{point,p}(q) G_{Ep}(q) + \frac{N}{Z} F_{point,n}(q) G_{En}(q) \right] F_{c.m.}(q), \tag{1}$$

where $F_{point,p}(q)$ and $F_{point,n}(q)$ are the form factors which are related to the point-like proton and neutron densities $\rho_{point,p}(\mathbf{r})$ and $\rho_{point,n}(\mathbf{r})$, respectively. These densities correspond to wave functions in which the positions $\mathbf{r}$ of the nucleons are defined with respect to the centre of the potential related to the laboratory system. In PWBA these form factors have the form

$$F_{point,p}(\mathbf{q}) = \frac{1}{Z} \int \rho_{point,p}(\mathbf{r}) e^{i\mathbf{qr}} d\mathbf{r} \quad \text{and} \quad F_{point,n}(\mathbf{q}) = \frac{1}{N} \int \rho_{point,n}(\mathbf{r}) e^{i\mathbf{qr}} d\mathbf{r}, \tag{2}$$

where

$$\int \rho_{point,p}(\mathbf{r}) d\mathbf{r} = Z; \quad \int \rho_{point,n}(\mathbf{r}) d\mathbf{r} = N. \tag{3}$$

In order that $F_{ch}(q)$ corresponds to density distributions in the centre-of-mass coordinate system, a factor $F_{c.m.}(q)$ is introduced in the standard way [$F_{c.m.}(q) = exp(q^2/4A^{2/3})$]. In Eq. (1) $G_{Ep}(q)$ and $G_{En}(q)$ are the Sachs proton and neutron electric form factors, correspondingly, and they are taken from one of the most recent phenomenological parametrizations [9].

In addition to PWBA, we also perform DWBA calculations solving the Dirac equation which contains the central potential arising from the proton ground-state distribution [6, 7, 8].

The theoretical predictions for the point-like proton and neutron nuclear densities of the light exotic nuclei ^{6,8}He and ^{11}Li, as well as of the corresponding stable isotopes ^{4}He and ^{6}Li are taken from the LSSM calculations. For ^{4,6,8}He nuclei they are obtained in a complete $4\hbar\omega$ shell-model space [10]. The LSSM calculations use a Woods-Saxon single-particle wave function basis for ^{6}He and ^{8}He and HO one for ^{4}He. The proton and neutron densities of ^{6}Li are obtained within the LSSM in a complete $4\hbar\omega$ shell-model space and of ^{11}Li in complete $2\hbar\omega$ shell-model calculations [11]. For ^{6}Li the single-particle HO wave functions have been used in the LSSM calculations and Woods-Saxon ones for ^{11}Li.

The density distributions of Ni, Kr and Sn isotopes are taken from deformed self-consistent HF+BCS calculations with density-dependent SG2 effective interactions using a large HO basis with 11 major shells [12, 13].

3 Results and Discussion

We calculate charge form factors for a variety of exotic nuclei with both PWBA and DWBA.

In Fig. 1 the point proton and matter density distributions (normalized correspondingly to Z and A) calculated with LSSM for the He and Li isotopes are shown. Matter distribution is taken to be $\rho_m(r) = \rho_{point,p}(r) + \rho_{point,n}(r)$. In addition, we give the "experimental" charge densities of ^{4}He [14, 15] and the "experimental" point-proton density of ^{6}Li [16]. One can see the considerable difference between the "experimental" charge densities of ^{4}He and the point proton densities calculated in LSSM. The calculated matter distributions for the halo nuclei are much more extended than the proton ones. The calculated matter densities for ^{8}He and ^{11}Li are in fair agreement with the experimental data obtained in proton scattering on these isotopes in GSI [17] (gray area).

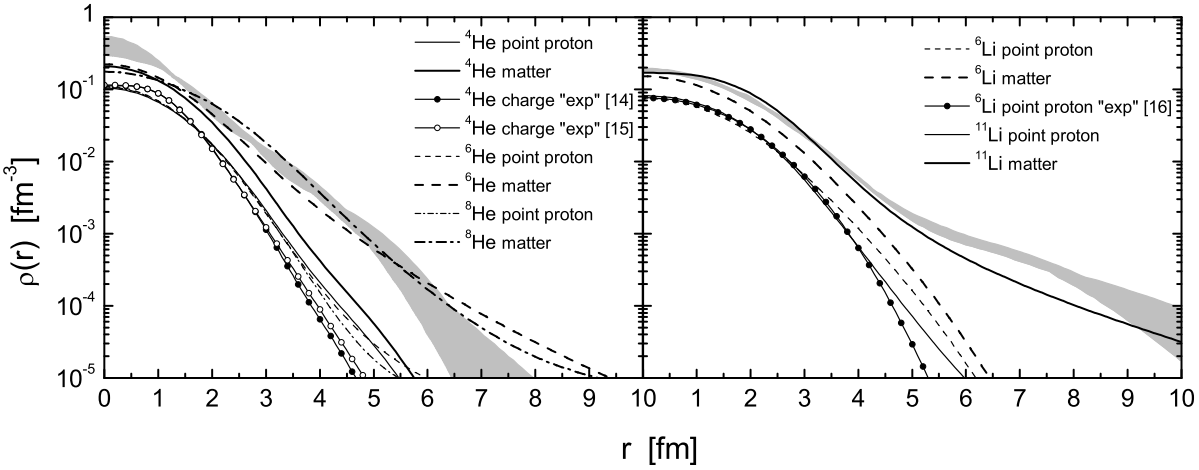

Figure 1: Thin lines are LSSM point proton densities of ^{4,6,8}He and ^{6,11}Li compared to the "experimental" charge density from "model-independent" analyses for ^{4}He [14, 15] and ^{6}Li [16]. Thick lines are LSSM matter densities of ^{4,6,8}He and ^{6,11}Li compared to matter density of ^{8}He and ^{11}Li deduced from the experimental proton scattering cross section data in [17] (grey area).

In Fig. 2 proton density distributions for isotopes of Ni, Kr, and Sn are compared. It can be seen that the point proton densities in a given isotopic chain decrease in the central region and increase in the surface with increasing neutron number.

The charge form factors for the He and Li isotopes in DWBA (at an energy of 540 MeV) and PWBA are shown in Fig. 3. The available experimental data for ^{4}He and ^{6}Li are also shown for comparison. One can see in the left panel the similarity of the LSSM charge form factors of ^{4}He and ^{6}He and their difference from that of ^{8}He and the small deviation of the DWBA from PWBA results in the whole q-range. At the same time there is not a minimum in this q-range in the form factors of ^{4,6,8}He and ^{6}Li in contrast to the experimental data of ^{4}He and ^{6}Li and this leads us to the conclusion that the LSSM densities of these light isotopes do not seem reliable.

In Fig. 4 we present the charge form factors calculated with DWBA at an energy of 250 MeV for the Ni, Kr, and Sn isotopes. A common feature of the charge form factors is the shift of the minima to smaller values of q when the number of neutrons increases in a given isotopic chain. This is due mainly to the enhancement of the proton densities in the peripherical region and also (to a minor extent) to the contribution of the charge distribution of the neutrons themselves. A common feature is also the expected filling of the Born zeros when DWBA is used (instead of PWBA), as well as the shift of the minima to smaller values of q and the increase of the

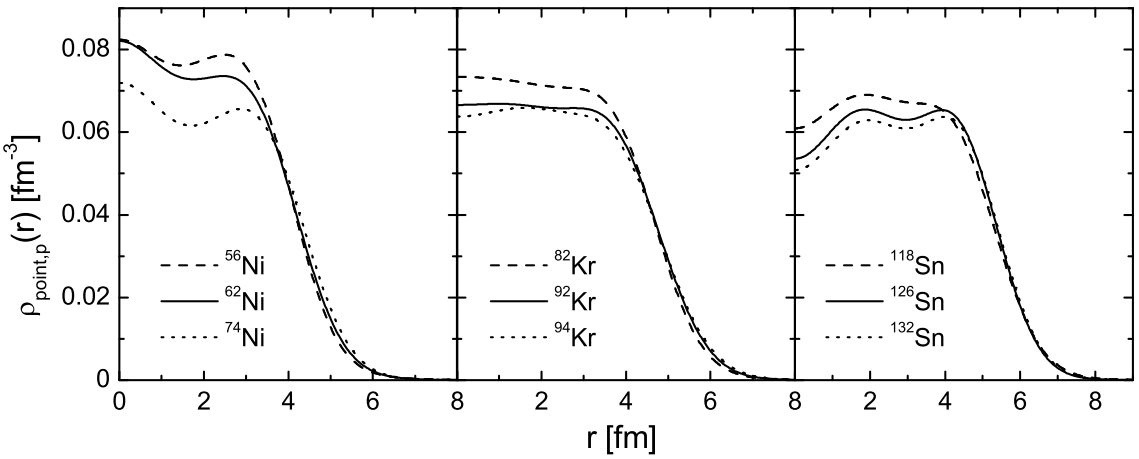

Figure 2: HF+BCS proton densities for isotopes of Ni, Kr, and Sn.

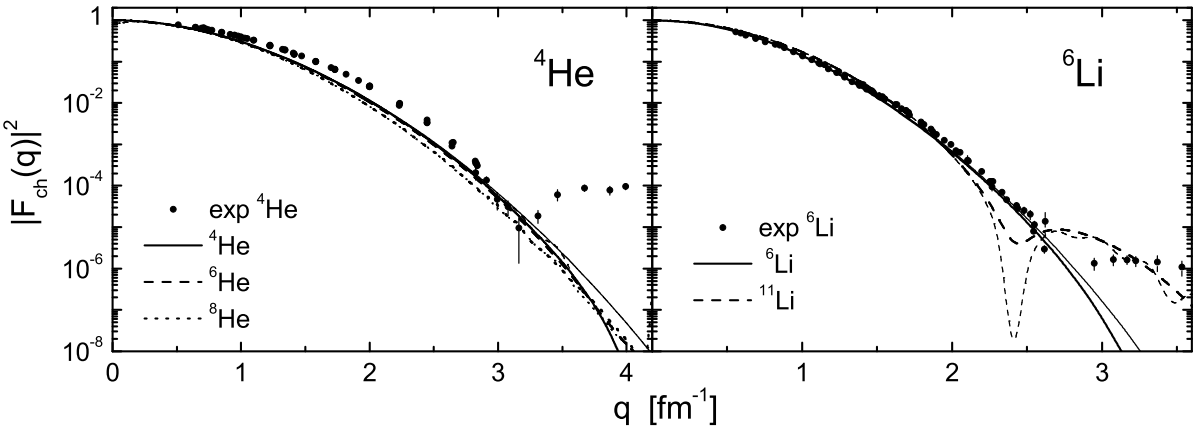

Figure 3: Charge form factors in DWBA (thick lines) and in PWBA (thin lines) for ^{4,6,8}He and ^{6,11}Li using LSSM densities and the experimental data for ^{4}He and ^{6}Li.

secondary peaks. We would like to note the reasonable agreement of the results of the DWBA calculations with the experimental charge form factors of ^{62}Ni and ^{118}Sn. The agreement with the empirical data for the stable isotopes is supportive of our results on the exotic nuclei to be used as guidance to future experiments.

In Table 1 we give the rms radii (R_p, R_n, R_{ch}, R_m) corresponding to nuclear proton, neutron, charge and matter distributions, as well as the difference $\Delta R = R_m - R_p$ for the He, Li, Ni, Kr and Sn isotopes which are considered in our work. For comparison we give additionally the charge and matter radii deduced from the electron and proton scattering experiments and from the total interaction cross sections. It is seen that the calculated charge rms radii of ^{4}He and ^{6}Li are larger than the experimental ones and the matter density of ^{11}Li exhibits the most extended halo component. The general trend of the difference ΔR between the matter and proton rms radii is to increase with the number of neutrons but for the heavy isotopes this increase is moderate compared to that of the light ones. The common tendency of all predicted rms radii for medium (Ni) and heavy (Kr and Sn) nuclei is the small increase of their values with the increase of the number of neutrons in a given isotopic chain except that R_{ch} of ^{126}Sn is practically the same as R_{ch} of ^{118}Sn. Our theoretical results on R_{ch} of Ni, Kr and Sn isotopes are in good agreement with the available experimental values.

The performed theoretical analyses of the densities and charge form factors can be a step

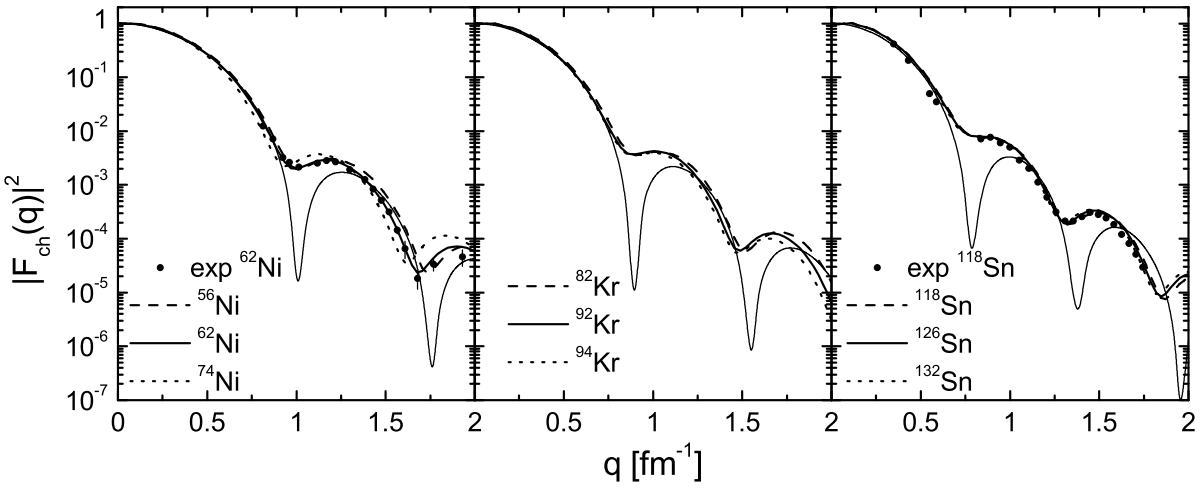

Figure 4: Charge form factors in DWBA for Ni, Kr, and Sn isotopes calculated using the HF+BCS densities and the experimental data for ^{62}Ni and ^{118}S. Thin solid lines are PWBA results for ^{62}Ni, ^{92}Kr and ^{126}Sn.

in the studies of the influence of the increasing neutron number on the proton and charge distributions in a given isotopic chain. This is important for understanding the neutron-proton interaction in the nuclear medium. We emphasize also the questions of interest, namely, the necessary both kinematical regions of the proposed experiments and precision to measure small shifts in the form factors.

The theoretical predictions for the charge form factors of exotic nuclei are a challenge for their measurements in the future experiments in GSI and RIKEN and thus, for obtaining detailed information on the charge distributions of such nuclei. The comparison of the calculated charge form factors with the future data will be a test of the corresponding theoretical models used for studies of the exotic nuclei structure.

The work was partly supported by the Bulgarian National Science Foundation under Contracts Φ-1416 and Φ-1501.

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Table 1: Proton (R_p), neutron (R_n), charge (R_{ch}), matter (R_m) rms radii (in fm) and difference $\Delta R = R_m - R_p$ of He and Li isotopes calculated using LSSM densities and Ni, Kr and Sn isotopes using HF+BCS densities. Available data on R_m and R_{ch} are also presented.

	R_p	R_n	R_{ch}	R_m	ΔR	R_m	R_{ch}
^{4}He	1.927	1.927	2.153	1.927	0.000	1.49(3) [18]	1.696(14) [14]
							1.695 [15]
^{6}He	1.945	2.900	2.147	2.621	0.676	2.30(7) [18]	
						2.33(4) [19]	
						2.54(4) [20]	
^{8}He	1.924	2.876	2.140	2.670	0.746	2.45(7) [18]	
						2.49(4) [19]	
^{6}Li	2.431	2.431	2.647	2.431	0.000	2.45(7)	2.57(10) [16]
						2.32(3) [19]	2.539 [15]
^{11}Li	2.238	3.169	2.477	2.945	0.707	3.62(19) [18]	
						3.12(16) [19]	
						3.53(10) [20]	
^{56}Ni	3.725	3.666	3.795	3.696	-0.029		
^{58}Ni	3.719	3.697	3.794	3.707	-0.012	3.764(10) [16]	
^{62}Ni	3.798	3.855	3.866	3.829	0.031	3.830(13) [16]	
^{74}Ni	3.911	4.130	3.977	4.049	0.138		
^{82}Kr	4.126	4.190	4.189	4.162	0.036		4.192(4) [21]
^{92}Kr	4.224	4.412	4.285	4.340	0.116		4.273(16) [21]
^{94}Kr	4.277	4.496	4.338	4.413	0.136		4.300(20) [21]
^{116}Sn	4.583	4.650	4.646	4.621	0.038	4.626(15) [16]	
^{118}Sn	4.649	4.739	4.705	4.701	0.052	4.679(16) [16]	
^{126}Sn	4.642	4.798	4.698	4.737	0.095		
^{132}Sn	4.685	4.879	4.740	4.807	0.122		

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