Resonance Analysis Combined with Optical Model

Toru Murata* and Tsuneo Nakagawa**
*former Toshiba, **JAEA Nuclear Data Center
e-mail: t.murata@ma.point.ne.jp

For the resonance analysis of neutron total and elastic scattering cross sections of light nuclides, combined model with the optical model is presented, and as an example, total cross section of the $^{18}$O+n reaction was analyzed.

1. Introduction

Cross sections of light nucleus show composite resonance structures in the incident energy range over 10 MeV. Ordinary method of resonance analysis requires frequently some wide resonance levels in the outside of the objective resonance region. These background resonance levels would not be explained physically. These background resonance levels could be predicted by the optical model, as some dispersed single particle states, using adequate potential parameters. So, combination of resonance formula and the optical model will be useful to analyze resonance region cross sections and effective to obtain continuation of nuclear data between resonance region and higher energy region that will be analyzed with the optical model.

2. Analysis method

In the present model, the collision matrix $U$ which describes the neutron elastic scattering channel is assumed to be given by the sum of optical model $U^{opt}$ and resonance formula $U^{res}$ of the same spin-parity state, such as

$$U_{J\pi} = kU^{opt}_{J\pi} + (1-k)U^{res}_{J\pi}$$

(1)

where $k$ is the optical model weight factor and resonance term weight was determined to hold $U$ unitary approximately and normalized so as to $|U_{J\pi}|^2 = 1$. The collision matrix of R-matrix is given by, under assumption that the same spin parity resonance levels have same ratio of reduced width between each channel$^3$. 

\[ U_{cc', J\pi}^{\text{res}} = \exp(i(\omega_c - \phi_c)) \left[ \delta_{cc'} + \frac{\sum \Gamma_{J\pi}^{1/2} \Gamma_{J\pi'}^{1/2} / (E_{J\pi} - E)}{1 + \sum \Delta_{J\pi} - i\Gamma_{J\pi} / 2 / (E_{J\pi} - E)} \right]_{J\pi} \exp(i(\omega_c - \phi_c)) \]  

(2)

where notations are given in article by Lane and Thomas\(^2\) and energy dependence of resonance widths are given by using barrier penetration factor \( P_t \)

\[ \Gamma_{J\pi}(E) = P_t(E) \Gamma_{J\pi}(E_{J\pi}) / P_t(E_{J\pi}) \]  

(3)

The optical model collision function \( U_{\text{opt}} \) is given by

\[ U_{\text{opt}}^{J\pi} = \exp(2i\delta_{J\pi}) \text{, and } \delta_{J\pi} = \alpha_{J\pi} + i\beta_{J\pi} \]  

(4)

Real and imaginary part of phase shift \( \delta_{J\pi} \) and \( \delta_{J\pi} \) are calculated with optical model code such as ELIESE-33.

Cross sections of total and elastic scattering and angular distribution of scattered neutrons are calculated using the combined collision function given by Eq. (1) with ordinary formula\(^2\):

Total cross section

\[ \sigma_{\text{tot}} = \frac{\pi}{k^2_n} \sum_{J\pi} 2g_J [1 - \text{Re}(U_{J\pi}^2)] \]  

(5)

Elastic scattering cross section

\[ \sigma_{\text{el}} = \frac{\pi}{k^2_n} \sum_{J\pi} g_J [1 - \text{Re}(U_{J\pi}^2) + \text{Im}(U_{J\pi}^2)] \]  

(6)

Angular distribution (general case)

\[ d\sigma_{\text{av}} = \sum_{s, \tau} \frac{k_n^{-2}}{(2s + 1)} \sum_L B_L(a's'; as, \tau) P_L(\cos \theta)d\Omega \]  

(7)

\[ B_L(a's'; as, \tau) = \frac{(-1)^{s-t}}{4} \sum_{l_1, l_2, l_3, l_4} i^{l_1-l_2-l_3-l_4} Z(l_1, J_1, l_2, J_2, sL) \times l^{l_1-l_2-l_3-l_4} Z(l_1', J_1', l_2', J_2', s' L) \times \text{Re}[(\delta_{a'as'l_1,l_2',J_1,J_2})^* (\delta_{a'a'l_1',l_2',J_1',J_2'})] \]

3. Result of application and discussion

We are now evaluating the nuclear data for the \(^{18}\text{O}+\text{n}\) reaction and analyzing the experimental total cross sections measured by Vaughn et al.\(^4\) and by Salisbury et al.\(^5\). Koehler et al.\(^6\) measured angular distributions of elastic and inelastic (\(E_X=1.98\text{MeV}\)) scattered neutrons and the analysis was made in detail with a multilevel-multichannel \(R\)-matrix code and resonance parameters were obtained in the incident neutron energy region
$E_n = 0 \sim 7.5 \text{ MeV}$. For the analysis, they assumed seven broad background resonance levels in the region $E_n \approx 10 \sim 15.5 \text{ MeV}$. To examine the effect of the background resonance levels, we reanalyze the total cross section below $E_n=5.0 \text{ MeV}$ with the resonance formula given in Eq.(2) by adjusting the resonance parameters given by Koehler et al.. Result of the reanalysis is shown in Fig.1. If no background resonance levels included, calculated cross section (dashed line) is fairly larger than experimental one. The background resonance levels seem to be set to reproduce the average experimental cross sections.

![Figure 1](image)

Fig.1  Effect of the seven background resonance levels assumed in $E_n>10\text{MeV}$ by Koehler et al.$^6$ Solid line shows the calculated cross section using full resonance levels and dashed line shows calculated one without the background resonance levels.

Figure 2 shows the total cross section in the incident energy region $E_n<=5.0 \text{ MeV}$ with the present model comparing with the experimental data. The dashed line shows the optical model total cross section calculated with ELIESE-3 code using potential parameters given by Wilmore and Hodgeson$^7$. The optical model $U^{\text{opt}}$ was calculated with the same code and same potential parameters. The resonance $U^{\text{res}}$ was calculated using 14 resonances of almost same resonance parameters of the calculation in Fig.1 and the optical model weight factor $k=0.35$.

Though the present model will be applied to analyze the cross section of resonance region without background resonance levels, simple weighted sum of collision functions given by Eq. (1) will reduce somewhat pure resonance amplitude and there is possibility to mis-assign the spin-parity of resonance levels. Further study shall be made for the combination of the optical model and resonance formula from more fundamental stand points.
Fig. 2 Comparison between the experimental cross sections and those calculated with the present model (solid line) and with the optical model (dashed line).

References
3) Igarasi, S. JAERI 1224 (1972)