共鳴公式とパラメータ評価

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原研リニアックとORELAでの経験

平成18年2月1日 核データ・チュートリアル 原子力機構先端基礎交流棟大会議室

原子力機構 水本 元治

講義内容



中性子反応と断面積





核変換(捕獲、吸収、分裂)



反応の前後で保存

反応の収量

$$\mathcal{X}_{ab}d\ell = N_{x}d\ell \cdot n_{a} \cdot \sigma_{ab}\left[s^{-1}cm^{-2}\right]$$

毎秒標的核1個当たりに起こる反応の数 断面積 σ_{ab} = 毎秒1cm²当たりに入射する粒子の数 エネルギー(質量を含む)、運動量、 角運動量、電荷、核子数、パリティー 面積の次元(cm²)を持つ。通常10⁻²⁴cm²(バーン)



¹⁴⁹Smとウランの中性子断面積

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• Breit-Wigner 公式 G. Breit and E.P. Wigner Phys. Rev., 49, 519 (1936). $\sigma_{n,x}(E) = \frac{\pi}{k^2} \sum_{l} \sum_{j} g_{j} \sum_{r} \frac{\Gamma_{nr} \Gamma_{xr}}{(E - Er')^2 + \frac{1}{4} \Gamma_{r}^2}$ • 共鳴パラメータ JとLごとにE', Γ_n , Γ_x を評価

Eugene Paul Wigner (November 17, 1902 – January 1, 1995) was a Hungarian physicist and mathematician who received the Nobel Prize in Physics in 1963 "for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles". In 1939 and 1940, Dr. Wigner played a major role for a Manhattan Project. In 1946, Wigner accepted a job as director of research and development at Clinton Laboratory (now Oak Ridge National Laboratory) in Oak Ridge, Tennessee. Wigner returned to teaching and research at Princeton University.



Eugene Wigner



Gregory Breit

Gregory Breit (July 14, 1899 – September 11, 1981) was an Russian-born American physicist, professor at universities in New York, Wisconsin, Yale, and Buffalo. Together with Eugene Wigner he gave a description of particle resonant states. During the early stages of the war, Breit was chosen to supervise the early design of the first atomic bomb during an early phase in what would later become the <u>Manhattan Project</u>.

http://en.wikipedia.org/wiki/Eugene_Wigner Gregory_Breit

2006.2.1

複合核準位 λ が形成される確率振幅 $F_{\lambda}(E)$

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全系の波動関数 準位λの波動関数

exp $(-iEt/\hbar)\Psi(E)$ exp $(-iWt/\hbar)X_{\lambda}$ 波動関数の時間によらない部分 $X_{\lambda}, \Psi(E)$



JENDL3.3の共鳴順位のデータの例

(natFeの全断面積)



JENDL3.3 Resonance Parameters of ¹⁰⁹Ag (Example)

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以下略

ΔΕ-ΙΔΕΔ

(共鳴間・反応間のInterference(干渉)がない場合)



中性子強度関数

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$$S_{l} = \frac{\left\langle g \Gamma_{n}^{l} \right\rangle}{(2l+1)D_{l}} = \frac{1}{(2l+1)\Delta E} \sum_{j} g_{j} \Gamma_{nj}^{l}$$

Reduced Neutron Width

$$\Gamma_{nj}^{l} = \sqrt{\frac{1eV}{E}} \frac{\Gamma_{nj}}{V_{l}}$$

Average level spacing D_l Spin statistical weight factor $g_J = \frac{2J+1}{(2i+1)(2I+1)} = \frac{2J+1}{2(2I+1)}$ S-wave

$$S_{0} = \frac{\left\langle g \Gamma_{n}^{0} \right\rangle}{D_{0}}$$
$$\Gamma_{nj}^{0} = \sqrt{\frac{1eV}{E}} \Gamma_{nj}$$
$$\left\langle g_{j} \Gamma_{nj} \right\rangle = \sqrt{E} \left\langle g_{j} \Gamma_{nj}^{0} \right\rangle = \sqrt{E} S_{0} D_{0}$$

Penetrability Factors for a Square Well Potential

| l | Vt |
|---|--|
| 0 | 1 |
| 1 | $k^{2}R^{2}/(1+k^{2}R^{2})$ |
| 2 | k ⁴ R ⁴ /(9+3k ² R ² +k ⁴ R ⁴) |
| 3 | k ⁶ R ⁶ /(225+45k ² R ² +6k ⁴ R ⁴ +k ⁶ R ⁶) |

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共鳴パラメータのコンピレーション

- Said. F. Mughabghab et al.: "Neutron Cross Sections, Vol. 1, Part A, Z=1-60", Academic Press (1981). "Neutron Cross Sections, Vol. 1, Part B, Z=61-100", Academic Press (1984)
- 2. S.I. Sukhoruchkin et al.: "Low Energy Neutron Physics", Landolt-Börnstein (1998)
- Said F. Mughabghab : "Atlas of Neutron Resonances : Resonance Parameters and Thermal Cross Sections. Part A: Z=1-50. Part B: Z=51-100(HRD)" /Publisher:Elsevier Published 2006/03 (出版予定) US\$314.00

参考書

J.A.Harvey ed.,"Experimental Neutron Resonance Spectroscopy", Publisher: Academic Press 1970

JAERI LINAC Resonance Experiments



The Oak Ridge Electron Linear Accelerator (ORELA)



Transmission

11 Flight Paths. 8-18, 20, 35, 40, 85, 150, and 200 m flight stations. ORELA ^{206,207,208}Pb, Capt., Trans. ¹⁵⁹Tb, Capt.

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150 MeV e⁻ linac.

Rate = 1 - 1000 Hz.

 $\Delta t = 2 - 30 \, \text{ns.}$

P < 60 kW.

¹³⁶Xe,Capt, Trans

全断面積の測定



全断面積(透過率測定Transmission)

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Fig. 16.10 Neutron transmission spectrum for a natural uranium sample. Numbers associated with the absorption dips in the spectrum are resonance energies in electron volt.

Transmission Experiment

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$$T_{trans} = \frac{(I_{in} - Bkg_{in}) / mon_{in}}{(I_{out} - Bkg_{out}) / mon_{out}} = exp(-n\sigma_t)$$

 $\sigma_t = -\frac{1}{n} log(T_{trans})$

 T_{trans} : Transmission σ_t : total cross section n:sample thickness

I_{in}: sample in counts I_{out}:sample out counts Bkg_{in}:Background (to sample in) Bkg_{out}:Background (to sample out) mon: monitor counts

Sample Changer to

compensate the neutron fluctuation



Black Resonance Filter Method for Background Determination

捕獲·散乱断面積測定



捕獲断面積測定



Fig. 16.11 The number of capture counts per channel for a 0.8 mm thick tantalum sample plotted aginst neutron time-of-flight. (Electron pulse width 0.5 μ sec, Channel width 0.5 μ sec, Flight path length 52 m).

散乱断面積測定

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Fig. 16.12 The number of scattering (below) and capture (above) counts per channel versus neutron timeof-flight for 0.026, 0.39 and 1.9 mm thick samples of tungsten. Resonance energies and the isotopes responsible for the resonances are shown.

核分裂断面積測定

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Resolution Broadening

CPAF-JAEA $\boldsymbol{E} = \frac{1}{2}\boldsymbol{m}\boldsymbol{v}^{2} = \frac{1}{2}\boldsymbol{m}\left(\frac{\boldsymbol{L}}{\boldsymbol{t}}\right)^{2} = \left(\frac{72.3 \times \boldsymbol{L}}{\boldsymbol{t}}\right)^{2}$ $m = 938(MeV) / (2,9979 \times 10^8 (m / s))^2$ $= 938 \times 10^{6} \times 1.602 \times 10^{-13} / (2,9979 \times 10^{8} (m / s))^{2} kg = 1.675 \times 10^{-27} kg$ Target $dE / E = \left((2dt / t)^{2} + (2dL / L)^{2} \right)^{1/2}$ (Moderator) Sample (Detector) $= \left((2dt / (72.3 * L) * \sqrt{E_n})^2 + (2dL / L)^2 \right)^{1/2}$ $f_{Lc}(E) \approx \frac{1}{\Delta_{Lc} \sqrt{\pi}} \int_{-\infty}^{\infty} dE'' \exp \left\{ -\frac{(E'' - E)^2}{\Delta_{Lc}^2} \right\} f(E'')$

 $\Delta_{Lc}^{2} = \frac{2}{3} E^{2} \left[\left(\Delta t_{c} / t \right)^{2} + \left(\Delta L / L \right)^{2} \right]$

Doppler Broadening





THERMAL CROSS SECTIONS

 $\begin{aligned} \sigma_{\gamma}^{0} &= 40140.\pm600. \ b \\ \sigma_{e}^{0} &= 197. \ b \\ \sigma_{coh}^{0} &= 87. \ b \\ \sigma_{incoh}^{0} &= 110. \ b \\ \sigma_{\alpha}^{0} &= 30.7\pm2.1 \ mb \\ \sigma_{\alpha}^{0} &= 5.5\pm0.4 \ mb \ [^{146}Nd^{4}] \ [0+] \\ \sigma_{\alpha}^{0} &= 23.5 \ mb \ [2+] \\ b_{coh}^{0} &= -24-111 \ fm \\ R' &= 8.3\pm0.2 \ fm \\ g_{\gamma} &= 1.7102 \end{aligned}$

RESONANCE PROPERTIES

 $\begin{array}{l} <\Gamma_{\gamma 0}>= \ 0.062\pm 0.002 \ {\rm eV} \\ {\rm D}_0 \ = \ 2.2\pm 0.2 \ {\rm eV} \\ {\rm S}_0 \ = \ 4.6\pm 0.6 \\ {\rm S}_1 \ = \ 0.3\pm 0.1 \\ {\rm S}_{\gamma 0}= \ 282\pm 28 \\ {\rm I}_{*}^{*} \ = \ 3390 \ {\rm b} \end{array}$

RESONANCE PARAMETERS

| $1'' = 7/2^-$ $\sigma_{\gamma}(+) = 39770.$ b | σ_{γ} | %Abn ,(−) = 5. b | = 13.9 $\sigma_{\gamma}(B) = 365.$ | S _n = 7985 | 5.5±0.7 keV |
|--|-------------------|----------------------|---|------------------------|-------------------------|
| $E_0 (eV)$ | J | $2g\Gamma_n \ (meV)$ | $\Gamma_{\gamma} (meV)$ | $2g\Gamma_n^0 \ (meV)$ | Γ_{α} (µeV) |
| -0.285 | 3 | | (62) | 0.277 | 0.28 ±0.12 |
| *0.0973±0.0002 | 4 | 0.600± 0.009 | 60.5± 0.6 | 1.92 ±0.03 | 0.037±0.010 |
| 0.872 ±0.003 | 4 | 0.835± 0.045 | 59.8± 1.0 | 0.894±0.048 | 0.023±0.006 |
| 4.95 ±0.03 | 4 | 2.40 ± 0.25 | 61 ± 3 | 1.08 ±0.11 | 0.025±0.007 |
| 6.43 ±0.05 | 3 | 0.63 ± 0.28 | 68 ± 5 | 0.25 ±0.11 | 0.085±0.023 |
| 8.95 ±0.07 | 4 | 11.1 ± 0.7 | 68 ± 4 | 3.71 ±0.23 | 0.036±0.009 |
| 12.05 ±0.08 | 3 | 1.6 ± 0.2 | 62 ± 6 | 0.46 ±0.06 | 0.19 ±0.05 |
| 14.91 ±0.10 | 4 | 5.0 ± 5.8 | 72 ± 8 | 1.61 ±0.05 | 0.05 ±0.02 |
| 15.85 ±0.10 | 3 | 0.32 ± 0.04 | | 0.08 ±0.01 | <0.48 |
| 17.1 ±0.1 | 4 | 2.30 ± 0.06 | 94 ± 5 | 0.556±0.015 | <0.075 |
| 23.2 ±0.1 | 4 | 1.00 ± 0.02 | 72 ± 9 | 0.208±0.004 | <0.07 |
| 24.6 ±0.1 | 4 | 0.35 ± 0.02 | | 0.071±0.004 | |
| 25.2 ±0.1 | 3 | 13.70 ± 0.17 | 86 ± 6 | 2.729±0.034 | 0.11 ±0.02 |
| 26.1 ±0.1 | 4 | 3.9 ± 0.2 | 90 ± 5 | 0.76 ±0.04 | 0.030±0.015 |
| 28.0 ±0.1 | 3 | 0.53 ± 0.03 | | 0.100±0.006 | 0.7 ±0.3 |
| 29.9 ±0.1 | 3 | 2.9 ± 0.1 | | 0.53 ±0.02 | 0.21 ±0.02 |
| 30.8 ±0.1 | 4 | 10.6 ± 0.2 | 73 ± 7 | 1.91 ±0.04 | 0.02 ±0.01 |
| 33.9 ±0.1 | 4 | 6.3 ± 0.2 | 67 ±12 | 1.08 ±0.03 | 0.017 |
| 40.2 ±0.1 | 3 | 23.8 ± 1.0 | 110 ±22 | 3.75 ±0.16 | 0.38 ±0.03 |
| 41.3 ±0.1 | 3 | 21.0 ± 1.3 | 109 ± 9 | 3.27 ±0.20 | 0.37 ±0.03 |
| 44.3 ±0.1 | 4 | 66.0 ± 1.3 | 115 ±10 | 9.92 ±0.20 | 0.03 ±0.01 |
| 45.1 ±0.1 | 4 | 11.0 ± 0.6 | 52 ±11 | 1.64 ±0.09 | 20.05 ±0.02 |
| 49.5 ±0.1 | 3 | 5.64 ± 0.18 | | 0.802±0.026 | 0.11 ±0.02 |
| 50.5 ±0.1 | 3 | 2.4 ± 0.2 | | 0.34 ±0.03 | 0.32 ±0.16 |
| 51.6 ±0.1 | 4 | 47 ± 2 | 77 ± 8 | 6.5 ±0.3 | 0.028±0.015 |
| 57.4 ±0.1 | 4 | 32.0 ± 0.8 | 128 ±12 | 4.22 ±0.11 | \$0.0± \$0.0 |
| 59.7 ±0.1 | 4 | 63.5 ± 2.0 | 123 ±12 | 8.22 ±0.26 | 0.01 |
| 60.9 ±0.1 | 3 | 4.1 ± 0.2 | | 0.53 ±0.03 | <0.04 |
| 62.1 ±0.1 | 4 | 39 ± 2 | 110 ±12 | 4.9 ±0.3 | |
| 64.8 ±0.1 | 4 | 50 ± 2 | | 6.2 ±0.2 | \$0.0± 80.0 |

¹⁴⁹Sm 共鳴パラメータ BNL-325

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Γ_{γ} : constant Γ_{n} is proportional to Sqrt(En)

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¹⁴⁷Sm and ¹⁴⁹Sm 共鳴解析



Fig. 1a. Example of resonance analysis of the transmission data for ¹⁴⁷Sm. The solid line is a multi-level Breit-Wigner fit.



Fig. 1b. Example of resonance analysis of the transmission data for ¹⁴⁷Sm. The solid line is a multi-level Breit-Wigner fit.



Fig. 2a. Example of resonance analysis of the transmission data for ¹⁴⁹Sm. The solid line is a multi-level Breit-Wigner fit.



Fig. 2b. Example of resonance analysis of the transmission data for ¹⁴⁹Sm. The solid line is a multi-level Breit-Wigner fit.

Nuclear Physics A357(1981)90

²⁰⁶Pb 全断面積と捕獲断面積



FIG. 1. Neutron capture yields (top part) and neutron transmission data (bottom part) obtained at flight paths of 40.12 m and 78.203 m, respectively.

ORELA Exp Phys. Rev. C19 (1979) 335

実験解析コード

| Atta-Harvey : | Trans. Single Level B-W | , Area Analysis, | Atta-Harvey | | |
|---------------|--|------------------|---------------------|--|--|
| SIOB: | Trans. Multi-Level B-W, | Shape Analysis | , de Saussure et al | | |
| TACASI: | Trans, Capt., Self Indication, Single Level B-W, | | | | |
| | | Shape&Area, | Frohner | | |
| LSFIT: | Cap. Single Level B-W, | Shape&Area, | Macklin | | |
| SAMMY: | Trans. Capt. Fission etc. | , Reich Moore, | Larson | | |
| | | | | | |

Resonance-resonance interference Resonance self-shielding Doppler broadening Multiple scattering Experimental resolution broadening

Normalization, Background Fitting

Sammy R-M、MLBW

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Reich-Moore 公式
C.W. Reich and M.S. Moore Phys. Rev., 111, 929
(1958)
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In the MLBW approximation, all off-diagonal elements of level matrix A are neglected.

In the RM approximation, only those off-diagonal elements arising from photon channels are neglected.

The MLBW formulation does not include level-level interference nor the multi channel features of RM

For isolated resonances of non-fissile nuclei, the values for these parameters would be equal in the two cases.

Area Analysis

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²³⁸U 66.01eV



Area Analysisの例



M.Ohkubo J.Nucl.Sci.Technol.16(1979)701

SIOB Analysis(サンプルの厚さによる効果)





SIOB Analysis for ¹⁰⁷Ag and ¹⁰⁹Ag

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For analyses, $\Gamma\gamma$ =130meV for ¹⁰⁷Ag $\Gamma\gamma$ =140meV for ¹⁰⁹Ag are assumed

Cumulative Level

0.25

1.0 147Sm 0.8 ∑g Γn° (meV) 0.6 $10^{4}S_{o} = 4.8$ 0.4 0.2 а °ò 400 800 1200 1600 2000 Neutron Energy (eV)

Fig. 4a. $g\Gamma_n^{0}$ versus energy for levels in ¹⁴⁷Sm. The slope gives the s-wave strength function.



¹⁴⁹Sm 0.20 0.15 ∑grno (meV) 0⁴S_o=4.6 10⁴S₀= 3.7 0.10 0⁴ S_o= 12.8 0.05 b $0^4 S_0 = 2.2$ 0 100 200 300 400 500 0 Neutron Energy (eV)





P-波強度関数







Level Missing for ¹⁰⁷Ag and ¹⁰⁹Ag

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Moore procedure for missing resonances using the moments of the reduced neutron width distribution. truncated Porter Thomas distribution

$$(\sum \omega_i) (\sum \omega_i g \Gamma_{ni}^0) / (\sum \omega_i \sqrt{g \Gamma_{ni}^0})^2$$

Weight (ω_i) for -wave contribution is calculated by Bayes theorem

D=20eV for ¹⁰⁷Ag D=20eV for ¹⁰⁹Ag

Porter Thomas Distribution



Fig. 5b. Histograms of observed $(g\Gamma_n^{0} \text{ MeV})^{1/2}$ values for ¹⁴⁹Sm The results for two choices of upper limits are shown.

Measured at 55m flight path

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$$P(x) = \frac{1}{\sqrt{2\pi x}} e^{-\frac{x}{2}}$$
$$x = \Gamma_{n0} / \langle \Gamma_{n0} \rangle$$

Chi-squared distribution with the degree of freedom v=1

NP, A357(1981)90

Wigner Distribution



Measured at 190m flight path

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$$\boldsymbol{P}(\boldsymbol{D}) = \frac{\pi}{2} \left(\frac{\boldsymbol{D}}{\langle \boldsymbol{D} \rangle^2} \right) \boldsymbol{e}^{-\pi \left(\frac{\boldsymbol{D}}{2 \langle \boldsymbol{D} \rangle} \right)^2}$$

The behavior of the eigenvalues of a symmetric matrix with random Gaussian distribution



放射幅の分布
$$P(x,v) = \frac{\frac{n}{2}}{\Gamma\left(\frac{v}{2}\right)} \left(\frac{v}{2}x\right)^{\left(\frac{v}{2}-1\right)} e^{-\left(\frac{v}{2}\right)x}$$

P-波の平均放射幅









Different sample thickness





SIOB ¹⁴⁸Sm



55m flight path

Maximum energy 8.4keV

JAERI memo 61-048 (1986)

Tb Capture Cross Section



FIG. 1. The effective capture cross section for a 0.859 g/cm^2 thick metal sample of 159 Tb (solid curve) is generated from the least squares fitting program (see Table I) with Doppler broadening, resonance self-protection, multiple scattering, and Gaussian resolution.

Transmission of ¹³⁶Xe+n



¹³⁶Xe

93.6% enrich

SAMMY Analysis

(a) The resonance near 18.4 keV is not larger than the resolution of 0.1%. The curve shows a $p_{1/2}$ assignment and gives a better fit than $p_{3/2}$. The ¹³⁴Xe resonance near 17.7 keV has a width exceeding the resolution. The peak cross section is proportional to the statistical weight factor which gives a unique p3/2 assignment.

(b) The strong resonance near 480 keV could not be fitted with a single $p_{3/2}$ resonance.

(c) It was necessary to introduce a $p_{1/2}$ resonance with the same energy to obtain the reproduction of the transmission dip.

Phys. Rev. C31 (1985) 2041

The 137 Xe neutron resonances can be excited through beta decay from 137 I.

Sammy R-M、MLBW



共鳴領域の核分裂断面積



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Minor Actinide





全立体角型多重ガンマ線検出装置

LLFP Long Lived Fission Product



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