

# Effect of Neutron Anisotropic Scattering in Fast Reactor Analysis

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Numerical tests were performed about an effect of a neutron anisotropic scattering on criticality in the Sn transport calculation. The simplest approximation, the consistent P approximation and the extended transport approximation were compared with each other in one-dimensional slab fast reactor models. JAERI fast set which has been used for fast reactor analyses is inadequate to evaluate the effect because it doesn't include the scattering matrices and the self-shielding factors to calculate the group-averaged cross sections weighted by the higher-order moment of angular flux. In the present study, the sub-group method was used to evaluate the group-averaged cross sections. Results showed that the simplest approximation is inadequate and the transport approximation is effective for evaluating the anisotropic scattering.

## I Introduction

A neutron transport equation in a whole core has been usually solved by the discrete ordinate method (Sn method) in the fast reactor analyses in Japan. In the Sn method, a neutron anisotropic scattering has been considered by the transport approximation, in which the total cross sections are replaced by the transport cross sections. The transport approximation makes it possible to evaluate the  $P_1$  effect of an anisotropic scattering without an increase of computational burden in comparison to a calculation with an isotropic scattering. When higher-order anisotropic scattering is necessary to be considered, the simplest approximation, in which angular flux is assumed to be separable into the energy-dependent and the angle-dependent functions, is used. It has not been shown clearly that these approximations are adequate for the evaluation of the neutron anisotropic scattering.

In the present paper, we reviewed the method to evaluate the neutron anisotropic scattering with some papers<sup>(1, 2)</sup> and extracted a problem caused by a limitation of the JAERI Fast Set which has been used in fast reactor analyses in Japan. After that, we showed another approach to evaluate the anisotropic scattering and carried out numerical tests in simple fast reactor models.

## II Review of Theory

For simplicity, a one-dimensional slab system is considered. A static neutron transport equation is described as below.

$$\mu \frac{d\phi(x, \mu, E)}{dx} + \Sigma_t(x, E)\phi(x, \mu, E) = \int d\mu' \int dE' \phi(x, \mu', E') \Sigma_s(x, \mu' \rightarrow \mu, E' \rightarrow E) + Q(x, \mu, E) \quad (1)$$

Scattering cross section can be shown as

$$\Sigma_s(x, \mu' \rightarrow \mu, E' \rightarrow E) = \Sigma_s(x, \mu_0, E' \rightarrow E) \quad (2)$$

where  $\mu_0$  means the cosine of the scattering angle. After scattering cross sections and angular flux are extended by the Legendre polynomials, Eq.(1) is transformed to

$$\mu \frac{d\phi(x, \mu, E)}{dx} + \Sigma_t(x, E)\phi(x, \mu, E) = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} P_l(\mu) \int dE' \phi_l(x, E') \Sigma_{s,l}(x, E' \rightarrow E) + Q(x, \mu, E). \quad (3)$$

When Eq.(3) is averaged within an energy group, total cross section is defined as

$$\Sigma_{t,g}(x, \mu) = \frac{\int_{E \in g} \Sigma_t(x, E)\phi(x, \mu, E)dE}{\int_{E \in g} \phi(x, \mu, E)dE}. \quad (4)$$

This means that the group-averaged total cross section should be dependent on the angle. Since it is difficult to apply the angular-dependent total cross section to reactor calculations, several methods have been described in Ref.(1) and (2).

In the first method, it is assumed that the angular flux can be separated into the energy-dependent and the angular-dependent functions. Then group-averaged total cross sections can be calculated by using neutron flux as a weighting function and the cross sections become independent on the angle. Under the assumption, Eq.(3) can be rewritten as

$$\mu \frac{d\phi_g(x, \mu)}{dx} + \Sigma_{t,g}^0(x)\phi_g(x, \mu) = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} P_l(\mu) \sum_{g'=1}^G \phi_{l,g'}(x) \Sigma_{s,l,g' \rightarrow g}^l(x) + Q_g(x, \mu) \quad (5)$$

where  $\Sigma_{t,g}^0(x)$  means the group-averaged total cross section calculated by using the 0-th moment of angular flux as a weight and  $\Sigma_{s,l,g' \rightarrow g}^l$  means the  $l$ -th order of scattering cross section calculated by using the  $l$ -th moment of angular flux. In the present paper, this method is referred as "the simplest approximation".

The second method is to extend angular flux in left hand side of Eq.(3) by the Legendre polynomials. After the extension, Eq.(3) can be rewritten as

$$\mu \frac{d\phi_g(x, \mu)}{dx} = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} P_l(\mu) \sum_{g'=1}^G \phi_{l,g'}(x) \left[ \Sigma_{s,l,g' \rightarrow g}^l(x) - \Sigma_{t,g}^l(x) \delta_{gg'} \right] + Q_g(x, \mu). \quad (6)$$

After adding  $\Sigma_g(x)\phi_g(x, \mu)$  to both sides of this equation, Eq.(6) can be rewritten as

$$\begin{aligned} \mu \frac{d\phi_g(x, \mu)}{dx} + \Sigma_g(x)\phi_g(x, \mu) &= \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} P_l(\mu) \sum_{g'=1}^G \phi_{l,g'}(x) \\ &\times \left[ \Sigma_{s,l,g' \rightarrow g}^l(x) + (\Sigma_g(x) - \Sigma_{t,g}^l(x)) \delta_{gg'} \right] + Q_g(x, \mu). \end{aligned} \quad (7)$$

When  $\Sigma_g(x)$  is defined as  $\Sigma_{t,g}^0(x)$ , it is called as "the consistent P approximation". If  $\Sigma_{t,g}^0(x)$  is assumed to be equal to  $\Sigma_{t,g}^l$ , Eq.(7) is coincident with Eq.(5).

If we consider the anisotropic scattering up to the  $P_L$  order in Eq.(7), an ignorance of the higher-order than  $P_{L+1}$  causes an error. The term of  $P_{L+1}$  is

$$\frac{1}{4\pi} (2L+3) P_{L+1}(\mu) \sum_{g'=1}^G \phi_{L+1,g'} \left[ \Sigma_{s,L+1,g' \rightarrow g}^{L+1}(x) + ((\Sigma_g(x) - \Sigma_{t,g}^{L+1}(x)) \delta_{gg'}) \right]. \quad (8)$$

To minimize the term, an approximation is used.

$$\sum_{g'}^G \phi_{L+1,g'} \Sigma_{s,L+1,g' \rightarrow g}^{L+1}(x) \approx \sum_{g'}^G \phi_{L+1,g'} \Sigma_{s,L+1,g \rightarrow g'}^{L+1}(x) \quad (9)$$

The cross section  $\Sigma_g(x)$  can be defined to minimize the error as below.

$$\Sigma_g(x) = \Sigma_{t,g}^{L+1}(x) - \sum_{g'=1}^G \Sigma_{s,L+1,g \rightarrow g'}^{L+1}(x) \quad (10)$$

This is the third method, called as "the extended transport approximation".

In the present paper, these described methods, the simplest approximation, the consistent P approximation and the extended transport approximation, were used to evaluate the anisotropic scattering in Sn calculations.

### III Application

Usually, JAERI Fast Set(JFS) has been used for fast reactor analyses in Japan. JFS includes the infinite-dilution cross sections, the scattering matrices of the  $P_0$  component and the self-shielding factors. The self-shielding factors are implemented to calculate the flux-weighted group-averaged cross sections. In addition, the current-weighted group-averaged cross section can be calculated for only the total reaction because it is necessary to define the transport cross section in the transport approximation. When we evaluate the anisotropic scattering with the described methods, it is necessary to evaluate the group-averaged cross section weighted by the higher-order moment of angular flux. The scattering matrices of the higher order components were given in the new type of the JFS library under a development in JNC. Therefore, we have to add "the higher order moment-weighted" self-shielding factors to the new JFS. But it needs much works to calculate the self-shielding factors and to reconstruct the format of the new JFS.

This problem can be overcome easily by introducing the sub-group method(3). In the sub-group method, a discretization is carried out with not energy but total cross section. The  $n$ -th order moment of angular flux  $\phi_n(E)$  can be expressed as below in the large homogeneous medium by the  $B_N$  method.

$$\phi_n(E) \propto \frac{1}{[\Sigma_t(E)]^{n+1}} \quad (11)$$

Group-averaged cross section weighted by the  $n$ -th order moment,  $\sigma_{x,g}^n$ , is calculated as

$$\sigma_{x,g}^n = \frac{\int_{E \in g} dE \sigma_x(E) \phi_n(E)}{\int_{E \in g} dE \phi_n(E)} = \frac{\int_{E \in g} dE \sigma_x(E) \frac{1}{[\Sigma_t(E)]^{n+1}}}{\int_{E \in g} dE \frac{1}{[\Sigma_t(E)]^{n+1}}} = \frac{\int_{E \in g} dE \sigma_x(E) \frac{1}{[\sigma_t(E) + \sigma_0]^{n+1}}}{\int_{E \in g} dE \frac{1}{[\sigma_t(E) + \sigma_0]^{n+1}}} \quad (12)$$

$\sigma_0$  means the background cross section. A discretization of Eq.(12) is carried out with the sub-group method as below.

$$\sigma_{x,g}^n = \frac{\sum_{b=1}^B \sigma_{x,b} \frac{P_b}{(\sigma_{t,b} + \sigma_0)^{n+1}}}{\sum_{b=1}^B \frac{P_b}{(\sigma_{t,b} + \sigma_0)^{n+1}}} \quad (13)$$

The sub-group parameters,  $P_b$ ,  $\sigma_{x,b}$  and  $\Sigma_{t,b}$ , can be prepared by the MOMENTOF code(4) developed by Japan Nuclear Cycle Development Institute. The parameters are defined to preserve the cross section moment(5) in the code. In unresolved resonance energy region, TIMS-1 code(6) is utilized to make a "ladder" of resonance.

## IV Numerical tests and results

To evaluate the effect of the anisotropic scattering, one-dimensional slab fast reactor models were constructed. We prepared three homogeneous mediums, fuel, blanket and reflector, whose number densities are shown in Table 1 and constructed three models using these mediums as shown in Table 2. Model 1 is a conventional fast reactor, model 2 is a plutonium burning reactor and model 3 is a heterogeneously blanket-loaded reactor.

Sub-group parameters were prepared with the MOMENTOF code from JENDL-3.2 and  $\sigma_{x,g}^n$  was evaluated. An energy group structure was defined the same as JFS. Scattering matrices implemented in JFS were used. One-dimensional transport calculations were performed by the ANISN code(7). One mesh per 2.5 cm was given and the Sn order was set to be 16.

At first, we evaluated a sensitivity of a weighting function used for calculations of group-averaged scattering cross sections to  $k_{eff}$ . We used different methods for the calculations. The "correct" functions described in Eq.(11) were used in one method and the neutron flux was used commonly in the other method. Results obtained with the simplest approximation and the consistent P approximation are shown in Table 3. The sensitivity is the largest in a result of model 2 with the consistent P approximation and a difference between two weight functions is 100pcm. These results mean that it is adequate to use a neutron flux as a weighting function approximately for calculation of the higher-order scattering cross sections.

Next, comparisons between the simplest approximation and the consistent P approximation were carried out. Results are shown in Table 4. Differences are 300pcm in model 1, 130pcm in model 3 and 1000pcm in model 2. These were caused by the approximation to separate angular flux into energy-dependent and angular-dependent functions and the results showed that the error should not be ignored. The effects to consider the higher-order anisotropic scattering, a difference between the results of  $P_1$  and  $P_3$  calculations, were observed about 50pcm in model 2 and it was not so large.

Comparisons between the consistent P approximation and the extended transport approximation were also carried out. Results are shown in Table 5. Convergences of  $k_{eff}$ s were observed in both the approximations as an increase of considered  $P_L$  order and the converged values agreed with each other. Differences between  $k_{eff}$  obtained by the transport approximation and the converged one are 40pcm in model 1, 60pcm in model 2 and 20pcm in model 3. This results show that the transport approximation is adequate to evaluate the anisotropic scattering in the analyse of these simple models.

## V Conclusion

Several papers were reviewed and numerical tests were performed about a neutron anisotropic scattering. The simplest approximation, the consistent P approximation and the extended transport approximation were compared with each other in one-dimensional slab reactor models. The results showed that the transport approximation, which has been used for fast reactor analyses in Japan, is adequate for considering the anisotropic scattering. In the present study, the evaluations were performed in the simple reactor models. Therefore it is necessary to evaluate the effect in the more realistic model and we are going to perform it.

If the higher-order anisotropic scattering must be considered, the higher-order moment-weighted group-averaged cross sections are necessary. In the present paper, it is shown to be adequate to use a neutron flux as a weighting function approximately for calculations of the higher order scattering cross sections. However, higher-order moment-weighted total cross

sections are necessary because the simplest approximation should not be used. Therefore "the higher-order moment-weighted" self-shielding factors for the total reaction should be added to JFS library. Another candidate to overcome the problem is an introduction of the sub-group method. The sub-group method is very useful because of the flexibility in an evaluation of an in-group flux.

## References

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Table 1: Number densities of homogeneous mediums (Unit: $10^{24}atoms/cm^3$ )

	Fuel	Blanket	Reflector
U-238	$8.4 \times 10^{-3}$	$1.4 \times 10^{-2}$	
Pu-239	$9.6 \times 10^{-4}$		
Fe-Nat.	$1.1 \times 10^{-2}$	$6.2 \times 10^{-3}$	$5.3 \times 10^{-2}$
Cr-Nat.	$1.9 \times 10^{-3}$	$1.7 \times 10^{-3}$	$1.5 \times 10^{-2}$
Ni-Nat.			$6.7 \times 10^{-3}$
C	$1.1 \times 10^{-3}$		
O	$1.6 \times 10^{-2}$	$2.3 \times 10^{-3}$	
Na	$9.5 \times 10^{-3}$	$4.7 \times 10^{-3}$	

Table 2: Model configuration (Unit:cm)

	Model 1	Model 2	Model 3
Reflective boundary	0	0	0
Fuel region	0-42.5	0-35	0-22, 27.5-50, 55-77.5
Blanket region	42.5-57.5	35-65	22.5-27.5, 50-55, 77.5-92.5
Reflector region	57.5-72.5	-	92.5-107.5
Vacuum boundary	75.2	65	107.5

Table 3: Sensitivity of weighting function of scattering cross section on  $k_{eff}$ 

	Model 1		Model 2		Model 3	
	$\phi_0$	$\phi_l$	$\phi_0$	$\phi_l$	$\phi_0$	$\phi_l$
Simplest P0	1.02321		1.01773		1.01263	
Simplest P1	1.00488	1.00502	0.99904	0.99981	1.00341	1.00346
Simplest P3	1.00514	1.00528	0.99941	1.00017	1.00356	1.00362
Simplest P5	1.00514	1.00528	0.99941	1.00017	1.00356	1.00362
Consistent P1	1.00188	1.00204	0.98899	0.98996	1.00209	1.00215
Consistent P3	1.00217	1.00233	0.98944	0.99040	1.00225	1.00231

Table 4:  $k_{eff}$ s obtained by the simplest and the consistent P approximation

	Model 1		Model 2		Model 3	
	Simplest	Consistent	Simplest	Consistent	Simplest	Consistent
P0	1.02321		1.01773		1.01263	
P1	1.00502	1.00204	0.99981	0.98996	1.00346	1.00215
P2	1.00528	1.00234	1.00020	0.99041	1.00364	1.00233
P3	1.00528	1.00233	1.00017	0.99040	1.00362	1.00231
P3-P1 (pcm)	26	29	36	44	16	16

Table 5:  $k_{eff}$ s obtained by the consistent P and extended transport approximation

	Model 1		Model 2		Model 3	
	Consistent	Ext. Trans.	Consistent	Ext. Trans.	Consistent	Ext. Trans.
P0	1.02321	1.00275	1.01773	0.99101	1.01263	1.00251
P1	1.00204	1.00224	0.98996	0.99036	1.00215	1.00224
P2	1.00234	1.00233	0.99041	0.99037	1.00233	1.00231
P3	1.00233	1.00233	0.99040	0.99037	1.00231	1.00231